Mathematics

FINITE ELEMENT ANALYSIS AS COMPUTATION

What the textbooks don't teach you about finite element analysis

Chapter 9: Conclusion

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Chapter 9

Conclusion

9.1 Introduction

Technology and Science go hand in hand now, each fertilizing the other and providing an explosive synergy. This was not so in the beginning; as there was Technology much before there was Science. In fact, there was technology before there were humans and it was technology which enabled the human race to evolve; science came later. The FEM is also an excellent example of a body of knowledge that began as Art and Technology; Science was identified more slowly. This chapter will therefore sum up the ideas that constitute the science of the finite element method and also point to the future course of the technology.

9.2 The C-concepts: An epistemological summing-up

One would ideally like to find a set of cardinal or first principles that govern the entire finite element discretisation procedure, ensuring the quality of accuracy and efficiency of the method. At one time, it was believed that two rules, namely *continuity* and *completeness*, provided a necessary and sufficient basis for the choice of trial functions to initiate the discretisation process. Chapter 3 of this book reviewed the understanding on these two aspects. However, finite element practitioners were to be confronted with a class of problems where the strict implementation of the continuity condition led to conflicts where multiple strain fields were required. This was the locking problem which we examined carefully in chapters 5 to 7. Let us recapitulate our fresh insight in an epistemological framework below.

9.2.1 Continuity conflict and the consistency paradigm

We saw in chapters 5 to 7 that in a class of problems where some strain fields need to be constrained, the implementation of the continuity rules on the displacement trial functions blindly led to a conflict whereby excessive constraining of some strain terms caused a pathological condition called locking.

Let us again take up the simple problem of a Timoshenko beam element. Two strain energy components are important; the bending energy which is constituted from the bending strain as $\kappa = \theta,_x$ and the shear strain energy which is based on the shear strain $\gamma = \theta - w,_x$. The simplistic understanding that prevailed for a long time was that the existence of the $\theta,_x$ and $w,_x$ terms in the energy expression demanded that the trial functions selected for the θ and w fields must be C^0 continuous; i.e. continuity of θ and w at nodes (or across edges in a 2D problem, etc.) was necessary and sufficient.

However, this led to locking situations. To resolve this conflict, it became necessary to treat the two energy components on separate terms. The shear strain energy was constrained in the regime of thin beam bending to become vanishingly small. This required that the shear strains must vanish without producing spurious constraints. The idea of field consistent formulation was to anticipate these conditions on the constrained strain field. Thus for $\theta - w,_x$ to vanish without producing gratuitous constraints, the trial functions used for θ and w in the shear strain field alone must be complete to exactly the same order - this is the consistency paradigm. We can also interpret this as a multiple continuity requirement. Note that although the bending strain requires θ to be C^0 continuous, in the shear strain component, we require θ

not to have C^0 continuity, i.e. say a C^{-1} continuity, is. Thus, such a conflict is resolved by operating with consistently represented constrained strain fields.

9.2.2 The correctness requirement

The consistency requirement now demands that when trial functions are chosen for displacement fields, these fields (such as θ in $\gamma = \theta - w,_x$) which need to be constrained must have a reconstituted consistent definition. The minimum total potential principle, being a single field variational principle, does not provide a clue as to how this reconstitution of the strain fields can be performed without violating the variational rules.

In the course of this book, we have come to understand that a multi-field variational principle like the generalized Hu-Washizu theorem provided exactly that flexibility to resolve the conflicting demands between consistency and continuity. This, we called the *correctness* requirement - that the re-constitution of the strain field which needed a reduced-continuity satisfying displacement field from an inconsistent strain field kinematically derived from the original continuous displacement fields must be done strictly according to the orthogonality condition arising from the HW theorem.

9.2.3 The correspondence principle

The study so far with consistency and correctness showed the primacy of the Hu-Washizu theorem in explaining how the finite element method worked. It also became very apparent that if the internal working of the discretisation procedure is examined more carefully, it turns out that it is strain and stress fields which are being approximated in a "best-fit" sense and not the displacement fields as was thought earlier. This brings us to the stress correspondence paradigm, and the discovery that this paradigm can be axiomatised from the Hu-Washizu theorem, as shown in chapter 2.

9.3 How do we learn? How must we teach? In a chronological sense or in a logical sense?

In an epistemological sense, often, it is phenomena which first confronts us and is systematically recognized and classified. The underlying first principles that explain the phenomena are identified very much later, if not at all. The clear understanding of first principles can also lead to the unraveling of new phenomena that was often overlooked.

Thus, if and when the first principles are available, then the logical sequence in which one teaches a subject will be to start with first principles and derive the various facets of the phenomena from these principles. In pedagogical practice, we can therefore have a choice from two courses of action: teach in a chronological sense, explaining how the ideas unfolded themselves to us, or teach in the logical sense, from first principles to observable facts. It is not clear that one is superior to the other.

In my first book [9.1], I choose the chronological order. The conflicts created by the locking phenomena were resolved by inventing the consistency paradigm. This led to the correctness principle and the recognition that the Hu-Washizu principle formed a coherent basis for understanding the fem procedure. From these, one could axiomatise the correspondence rule - that the fem approach manipulates stresses and strains directly in a best-approximation sense. Thus, chapter 12 of Reference 9.1 sums up the first principles.

In the present book, I have preferred a logical sequence - the HW and correspondence rules are taken up first and then the various phenomena like locking, etc.

9.4 Finite element technology: Motivations and future needs

Advances in computer science and technology have had a profound influence on structural engineering, leading to the emergence of this new discipline we call *computational structural mechanics (CSM)*. Along with it a huge software industry has grown. CSM has brought together ideas and practices from several disciplines - solid and structural mechanics, functional analysis, numerical analysis, computer science, and approximation theory.

CSM has virtually grown out of the *finite element method (FEM)*. Algorithms for the use of the finite element method in a wide variety of structural and thermomechanical applications are now incorporated in powerful general purpose software packages. The use of these packages in the Computer-Aided-Design/Computer-Aided-Manufacturing cycle forms a key element in new manufacturing technologies such as the Flexible Manufacturing Systems. These allow for unprecedented opportunities for increase in productivity and quality of engineering by automating the use of structural analysis techniques to check designs quickly for safety, integrity, reliability and economy. Very large structural calculations can be performed to account for complex geometry, loading history and material behavior. Such calculations are now routinely performed in aerospace, automotive, civil engineering, mechanical engineering, oil and nuclear industries.

Modern software packages, called general purpose programmes, couple FEM software with powerful graphics software and the complete cycle of operations involving pre-processing (automatic description of geometry and subsequent sub-division of the structure) and post-processing (projecting derived information from FEM analysis on to the geometry for color coded displays to simplify interpretation and make decision making that much easier). Already artificial intelligence in the form of knowledge based expert systems and expert advisers and optimization procedures are being coupled to FEM packages to reduce human intervention in structural design to a bare minimum.

It is not difficult to delineate many compelling reasons for the vigorous development of CSM [9.2]. These are:

1. There are a large number of unsolved practical problems of current interest which still await experimental and/or numerical solutions. Some of these demand large computational power. Some of the examples described in reference 9.2 are: simulation of response of transportation vehicles to multidirectional crash impact forces, dynamics of large flexible structures taking into account joint nonlinearities and nonproportional damping, study of thermoviscoelatic response of structural components used in advanced propulsion systems etc. In many structural problems, the fundamental mechanics concepts are still being studied (e.g. in metal forming, adequate characterization of finite strain inelasticity is still needed).

2. Computer simulation is often required to reduce the dependence on extensive and expensive testing; in certain mission critical areas in space, computer modeling may have to replace tests. Thus, for large space structures (e.g. large antennas, large solar arrays, the space station), it may not be possible for ground-test technology in 1-g environment to permit confident testing in view of the large size of the structures, their low natural frequencies, light weight and the presence of many joints.

3. Emerging and future computer systems are expected to provide enormous power and potential to solve very large scale structural problems. To realize this potential fully, it is necessary to develop new formulations, computational strategies, and numerical algorithms that exploit the capabilities of these new machines (e.g. parallelism, vectorization, artificial intelligence).

Noor and Atluri [9.2] also expect that high-performance structures will demand the following technical breakthroughs:

1. Expansion of the scope of engineering problems modeled, such as:

   a) examination of more complex phenomena (e.g. damage tolerance of structural components made of new material systems);

   b) study of the mechanics of high-performance modern materials, such as metal-matrix composites and high-temperature ceramic composites;

   c) study of structure/media interaction phenomena (e.g. hydrodynamic/structural coupling in deep sea mining, thermal/control/structural coupling in space exploration, material/aerodynamic/structural coupling in composite wing design, electromagnetic/thermal/structural coupling in microelectronic devices);

   d) use of stochastic models to account for associated with loads, environment, and material variability

   e) development of efficient high-frequency nonlinear dynamic modeling capabilities (with applications to impulsive loading, high-energy impact, structural penetration, and vehicle crash-worthiness);

   f) improved representation of structural details such as damping and flexible hysteritic joints:

   g) development of reliable life-prediction methodology for structures made of new materials, such as stochastic mechanisms of fatigue, etc.

   h) analysis and design of intelligent structures with active and/or passive adaptive control of dynamic deformations, e.g. in flight vehicles, large space structures, earthquake-resistant structures

   i) Computer simulation of manufacturing processes such as solidification, interface mechanics, superplastic forming.

2. Development of practical measures for assessing the reliability of the computational models and estimating the errors in the predictions of the major response quantities.

3. Continued reduction of cost and/or time for obtaining solutions to engineering design/analysis problems.

Special hardware and software requirements must become available to meet the needs described above. These include:

1. Distributed computing environment having high-performance computers for large scale calculations, a wide range of intelligent engineering workstations for interactive user interface/control and moderate scale calculations.

2. User-friendly engineering workstations with high-resolution and high speed graphics, high speed long distance communication etc.

3. Artificial intelligence-based expert systems, incorporating the experience and expertise of practitioners, to aid in the modeling of the structure, the adaptive refinement of the model and the selection of the appropriate algorithm and procedure used in the solution.

4. Computerized symbolic manipulation capability to automate analytic calculations and increase their reliability.

5. Special and general purpose application software systems that have advanced modeling and analysis capabilities and are easy to learn and use.

9.5 Future directions

It is expected that CSM will continue to grow in importance. Three areas which are expected to receive increasing attention are: 1) modeling of complex structures; 2) predata and postdata processing and 3) integration of analysis programs into CAD/CAM systems.

The accurate analysis of a complex structure requires the proper selection of mathematical and computational models. There is therefore a need to develop automatic model generation facilities. Complex structures will require an enormous amount of data to be prepared. These can be easily performed by using predata and postdata processing packages using high resolution, high throughput graphic devices. The generation of three dimensional color movies can help to visualize the dynamic behavior of complex structural systems.

9.6 Concluding remarks

In this concluding chapter, we have summarized the ideas that should inform the rational development of robust finite elements and we have also described the current trends in the use of structural modeling and analysis software in engineering design. It has been seen that CSM has greatly improved our capabilities for accurate structural modeling of complex systems. It has the potential not only to be a tool for research into the basic behavior of materials and structural systems but also as a means for design of engineering structures.

9.7 References

9.1 G. Prathap, *The Finite Element Method in Structural Mechanics*, Kluwer Academic Press, Dordrecht, 1993.

9.2 A. K. Noor and S. N. Atluri, Advances and trends in computational structural mechanics, *AIAA J*, 25, 977-995 (1987).