APPLIED STATISTICS

Principles of acceptance sampling

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CONTENTS

Keywords

Acceptance Quality Level (AQL); Lot Tolerance Percent Defective (LTPD); Consumer's risk; Producer's risk; OC curve; ASN curve; ATI curve; AOQ curve; Rectifying inspection plans; Single sampling; Double sampling; Sequential sampling

Problem of lot acceptance

Another important aspect of statistical quality control is lot/ product control. However, from various practical and economic considerations it is not feasible to inspect each and every lot fully. The usual practice is to draw a random sample from the lot and take a decision about the lot disposition on the basis of sample fraction defective i.e. take recourse to sampling inspection. When a customer is offered a lot there are following three choices before him for accepting a lot:

a) without any inspection accept the lot
b) after 100% inspection accept the lot
c) select a fraction of the lot as sample and use an appropriate sampling plan

Sampling inspection has a number of advantages as:

a) it reduces time
b) it is applicable to destructive sampling
c) it reduces the inspection error considerably
d) it is less expensive
e) it reduces damage as there is lesser handling of the product.

When inspection is for the purpose of acceptance or rejection of the product based on certain quality standards the procedure employed is called Acceptance Sampling. Accepted sampling plans may be classified by attributes and variables. Here we shall talk of acceptance sampling plans for attributes first. It means that items will be judged as defective/ bad or non-defective/ good. Further, a sampling plan may be of either type viz. the acceptance-rejection type or acceptance-rectification type. In an acceptance-rejection sampling inspection plan lots are either accepted or rejected on the basis of the sample. In an acceptance-rectification sampling inspection plan if we do not accept on the basis of the sample, we take recourse to 100% inspection and in either case replace all defectives by non-defectives.

Before embarking upon a detailed discussion of the sampling inspection plans we shall first discuss a few concepts, which are quite important in their discussion:

Acceptance Quality Level (AQL)

This is the quality level of the supplier's process that the consumer would consider to be acceptable. Let p_1 be the fraction defective in a lot, which is fairly of good quality. Then

$$\text{P[rejecting a lot of quality } p_1] = 0.05$$

$$\Rightarrow \quad \text{P[accepting a lot of quality } p_1] = 0.95$$

p_1 is known as the acceptance quality level.

Lot Tolerance Percent Defective (LTPD)

It is the maximum fraction defective (p_t) in the lot that the consumer will tolerate. $100\,p_t$ is called lot tolerance percent defective. The probability of accepting lots with fraction defective p_t or greater is very small.

Average Outgoing Quality Limit (AOQL)

Let p be the fraction defective in the lot before inspection, also called the incoming quality. Then the expected number of fraction defectives remaining in the lot after the application of the sampling inspection plan is known as average outgoing quality. The maximum value of the AOQ, the maximum being taken with respect to p is called average outgoing quality limit.

Average Amount of Total Inspection (ATI)

The expected value of the sample size required for coming to a decision in an acceptance-rectification sampling inspection plan calling for 100% inspection of the rejected lots is called average amount of total inspection. It is a function of the incoming quality. The curve obtained on plotting ATI against p is called the ATI Curve.

Average Sample Number (ASN)

The expected value of sample size required for coming to a decision whether to accept or reject a lot in an acceptance- rejection sampling inspection plan is called ASN. The curve obtained on plotting ASN against p is called the ASN curve.

Producer's risk (P_p)

A producer can be an individual, a firm or a department that produces goods and supplies them to another individual, firm or department. As our decisions are based on sampling inspection plans he is always under the constant risk that sometimes certain lots of satisfactory quality be rejected. Further let the producer's average fraction defective be $\overline{p}$ i.e. a quality standard, which he has been able to maintain over a long period of time. Then probability of rejecting a lot of quality $\overline{p}$ is called producer's risk and is denoted by P_p.

$$\boxed{P_p = P[\text{rejecting a lot of quality } \overline{p}] = \alpha} \tag{1}$$

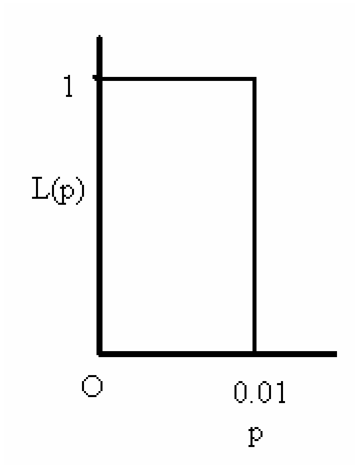
Consumer's risk (P_c)

By consumer here we will mean the person, firm or department that receives the articles from the producer. Just like producer, consumer is also faced with the risk of accepting a lot of unsatisfactory quality on the basis of sampling inspection. Thus probability of accepting a lot of quality p_t is called consumer's risk and is denoted by P_c.

$$\boxed{P_c = P[\text{accepting a lot of quality } p_t] = \beta} \tag{2}$$

Operating characteristic curve (OC)

The curve obtained on plotting the acceptance probability L(p) against p, the lot fraction defective is called OC curve. The discriminatory power of a sampling plan is revealed by its OC curve. The greater the slope of the OC. curve the greater the discriminatory power. By increasing the sample size we can increase the discriminatory power of the sampling plan. Of course, the acceptance number c should be kept proportional to n. The ideal OC curve looks like



However, this ideal OC curve can never be attained in reality.
There are basically two types of sampling plans:

1) Acceptance-rejection sampling plans and
2) Acceptance-rectification sampling plans or Rectifying sampling plans

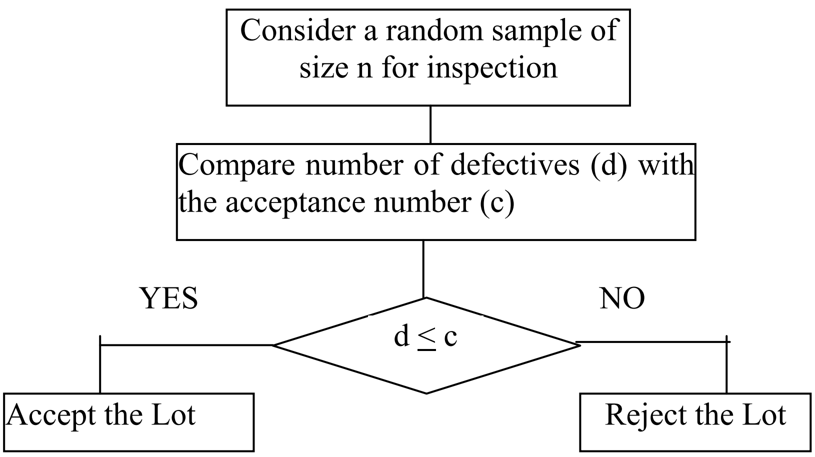
1. Acceptance – Rejection Sampling Plans

In these plans a decision to accept or reject a lot is taken on the basis of the samples drawn from it.

Single Sampling Plan

In this sampling plan, the decision to accept or reject the lot is based on a single sample. Draw a random sample of size n from the lot consisting of N units. If the number of defectives (d) in the sample is less than or equal to c accept the lot, otherwise reject it.

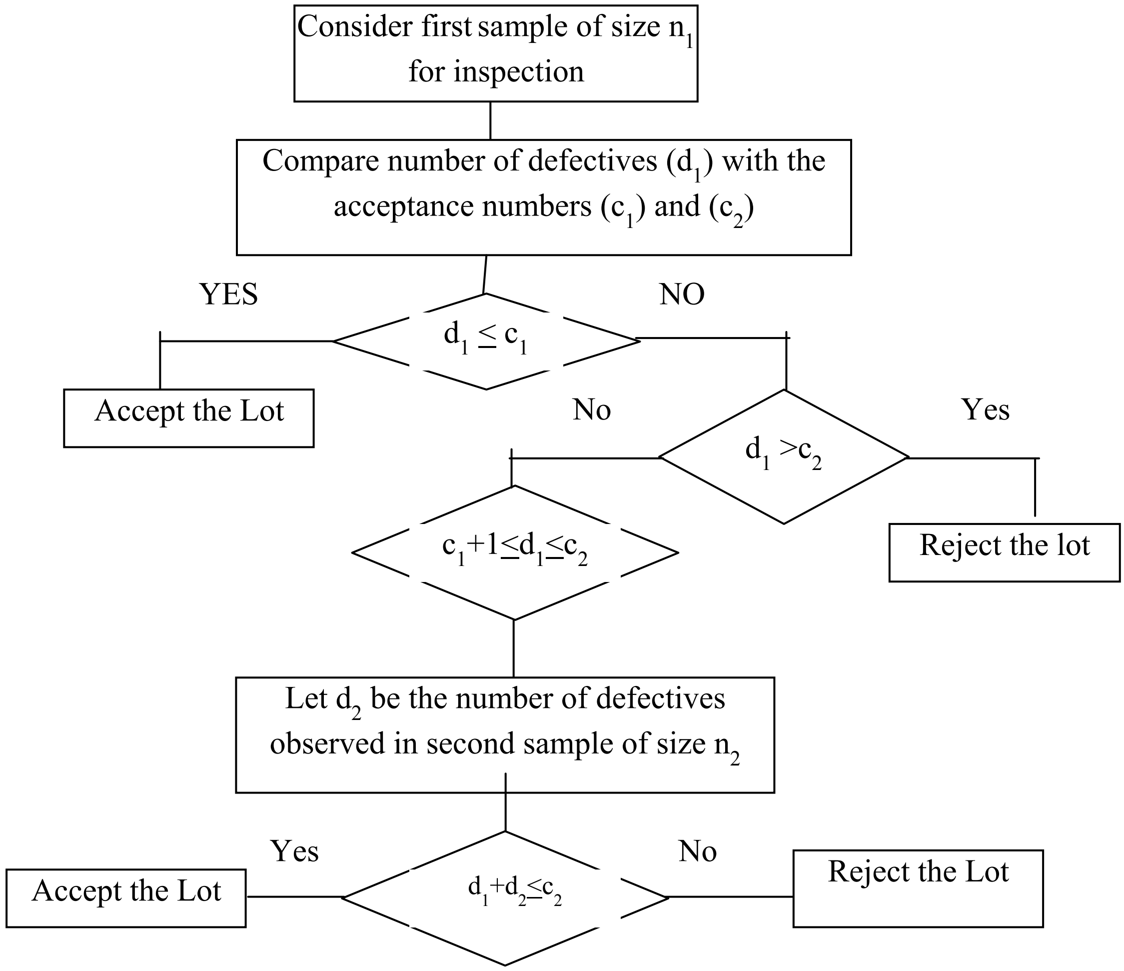
Decision Flow Chart for a Single Sampling Plan



Double Sampling Plan

Sometimes a second sample is required before one can reach a decision about acceptance or rejection of the lot. The procedure is: Draw a sample of size n_1 from the lot. If the number of defectives (d_1) obtained in the sample is $\leq c_1$ (sampling acceptance number for the first sample) accept the lot. If $d_1 > c_2$ (acceptance number for both the samples combined) reject the lot. However, if $c_1+1 \leq d_1 \leq c_2$, take another sample of size n_2. Let d_2 be the number of defectives observed in the second sample. If $d_1+d_2 \leq c_2$, accept the lot. If $d_1+d_2 > c_2$, reject the lot.

Decision Flow Chart for Double Sampling Plan



The Average Sample Number:

$$\boxed{ASN = n} \qquad (3)$$

as n units have to be inspected even if decision to accept or reject the lot is taken much before.

$$\boxed{\begin{aligned} ASN &= n_1P_1+(n_1+n_2)(1-P_1) \\ &= n_1+n_2(1-P_1) \end{aligned}} \qquad (4)$$

as only n_1 units will be inspected if the lot is accepted or rejected on the basis of first sample or (n_1+n_2) units will be inspected if a second sample has to be drawn.

Sequential Sampling Plan

It is an extension of the multiple sampling. Here units are selected from the lot one at a time, inspected and a decision is made either to accept the lot or reject the lot or select another unit.

2 Rectifying Sampling Inspection Plans

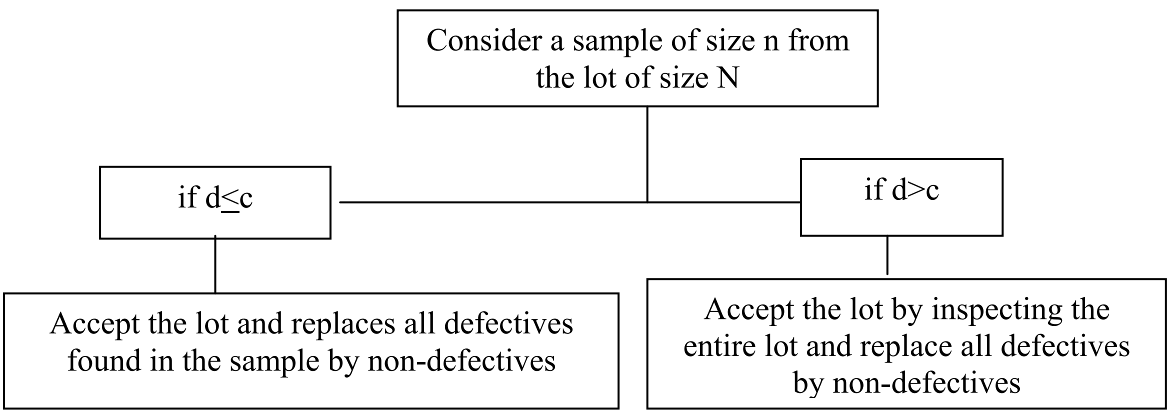
These plans were developed by Harold F. Dodge and G. Romig at the Bell Telephone Laboratories before World War II. Suppose that the incoming lots, which are being submitted for inspection have p_0 fraction defective. Some of these lots will be accepted on the basis of samples drawn from them, while others will be rejected. Under rectifying sampling inspection plans whenever we accept a lot we replace all the defective items encountered in the sample by good pieces. Whereas rejected lot are sent for 100% inspection and all the defectives encountered are replaced by good pieces. Thus these programs serve to improve lot quality.

For all the sampling inspection plans discussed earlier viz. single, double, sequential we simply need to interchange the phrases ‘accept’ and ‘reject’ by ‘accept’ and ‘replace all defective items in the sample’ and ‘inspect all the items in the lot and replace all defectives by good pieces’.

Rectifying Single Sampling Plan

Let p be the incoming quality, N the lot size, n the sample size, d the number of defectives in the sample and c the sampling acceptance number. Then the flow diagram for the above plan may be depicted as follows:

Flow Diagram for Rectifying Single Sampling Inspection Plan



Thus its **AOQ** will be given as:

$$\boxed{AOQ=\frac{(N-n)p.P_a(p)}{N}+0.[1-P_a(p)]} \qquad (5)$$

where, $P_a(p)$, is the probability of accepting a lot of quality p.

$$\boxed{P_a(p)=\sum_{x=0}^{c}\binom{Np}{x}\binom{N-Np}{n-x}\Big/\binom{N}{n}} \text{ (using hyper geometric distribution)} \qquad (6)$$

where, x, is the number of defectives observed in the sample.

The maximum ordinate on the AOQ curve representing the worst possible average quality that would result from the rectification inspection program, is called the AOQL.

The **Producer's Risk** is given by:

$$\boxed{P_p=1-\sum_{x=0}^{c}\binom{N\bar{p}}{x}\binom{N-N\bar{p}}{n-x}\Big/\binom{N}{n}} \qquad (7)$$

Next the **Consumer's Risk** is given by :

$$\boxed{P_c=\sum_{x=0}^{c}\binom{Np_t}{x}\binom{N-Np_t}{n-x}\Big/\binom{N}{n}} \qquad (8)$$

The **Average Amount of Total Inspection** is given by:

$$\boxed{ATI = n + (N-n)P_p} \qquad (9)$$

as n items have to be inspected in any case and the remaining (N-n) would be inspected if the number of defectives in the sample exceeds c.

OC curve of a single sampling plan:

Curved obtained on plotting $P_a(p)$ against p is called OC curve, where

$$\boxed{P_a(p)=\sum_{x=0}^{c}\binom{Np}{x}\binom{N-Np}{n-x}\Big/\binom{N}{n}} \qquad (10)$$

EXAMPLE – 1

In order to construct OC curve and to study ATI and AOQ curves a manufacturing unit selected a sample of size 150 from a lot of size 1800. Under single sampling plan a lot is accepted if the number of defective is less or equal to 12, 10, 5 & 1.

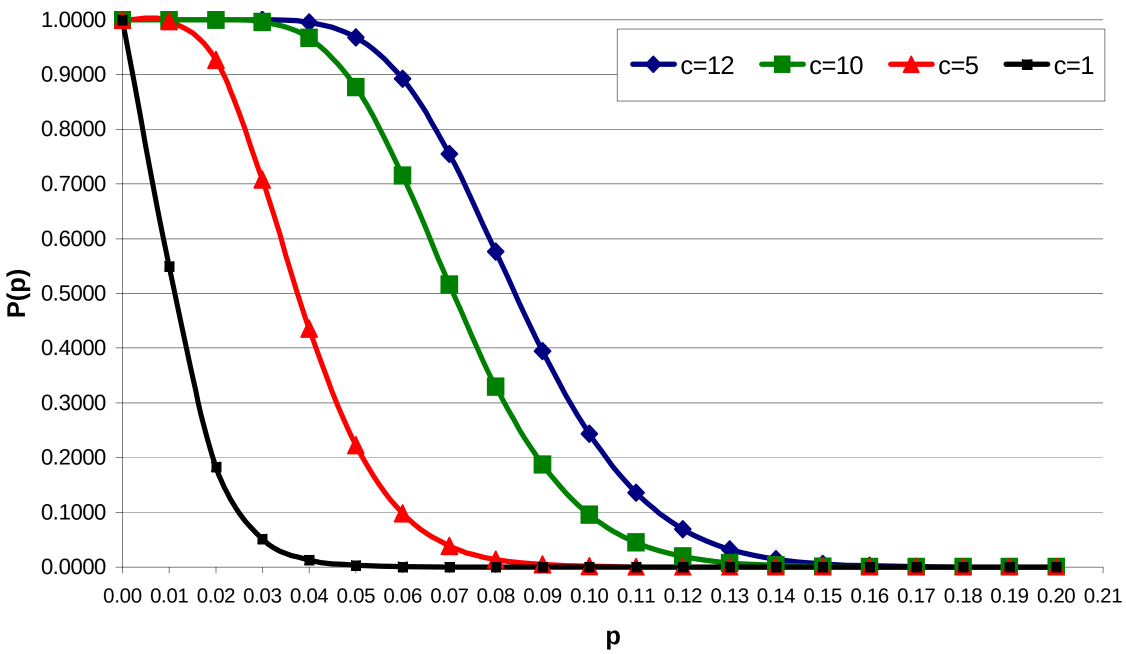
SOLUTION

The following table gives the values of $P_p(p)$, ATI and AOQ for different values of c.

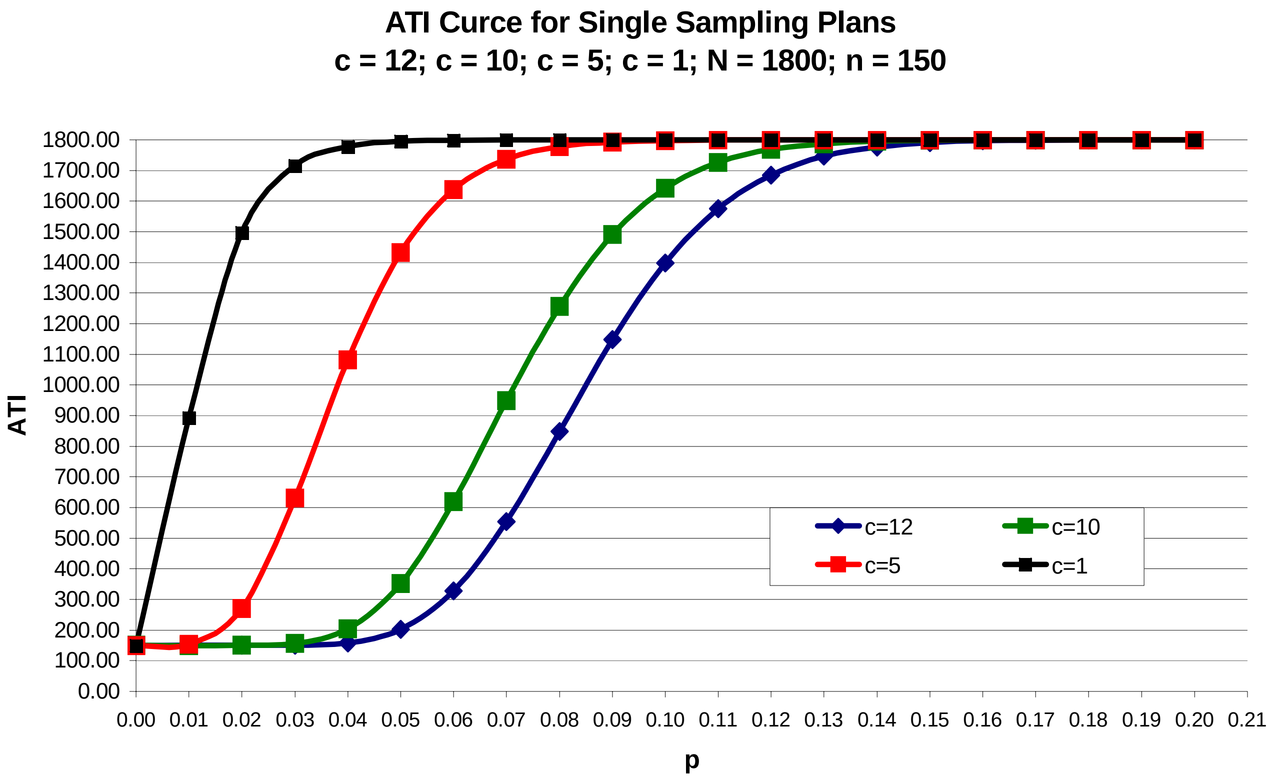
	$P_p(p)$	AOQ	ATI	$P_p(p)$	AOQ	ATI	$P_p(p)$	AOQ	ATI	$P_p(p)$	AOQ	ATI
p	c=12			c=10			c=5			c=1		
0.00	1.0000	0.0000	150.00	1.0000	0.0000	150.00	1.0000	0.0000	150.00	1.0000	0.0000	150.00
0.01	1.0000	0.0092	150.00	1.0000	0.0092	150.00	0.9976	0.0091	154.00	0.5498	0.0050	892.78
0.02	1.0000	0.0183	150.00	0.9999	0.0183	150.15	0.9269	0.0170	270.54	0.1834	0.0034	1497.35
0.03	0.9997	0.0275	150.50	0.9962	0.0274	156.33	0.7084	0.0195	631.07	0.0514	0.0014	1715.24
0.04	0.9948	0.0365	158.54	0.9674	0.0355	203.79	0.4355	0.0160	1081.41	0.0131	0.0005	1778.46
0.05	0.9681	0.0444	202.71	0.8774	0.0402	352.37	0.2227	0.0102	1432.60	0.0031	0.0001	1794.87
0.06	0.8922	0.0491	327.90	0.7155	0.0394	619.36	0.0982	0.0054	1637.89	0.0007	0.0000	1798.84
0.07	0.7551	0.0485	554.05	0.5161	0.0331	948.40	0.0385	0.0025	1736.47	0.0002	0.0000	1799.75
0.08	0.5765	0.0423	848.74	0.3293	0.0241	1256.67	0.0137	0.0010	1777.41	0.0000	0.0000	1799.95
0.09	0.3951	0.0326	1148.17	0.1873	0.0155	1490.90	0.0045	0.0004	1792.60	0.0000	0.0000	1799.99
0.10	0.2437	0.0223	1397.97	0.0960	0.0088	1641.55	0.0014	0.0001	1797.74	0.0000	0.0000	1800.00
0.11	0.1361	0.0137	1575.38	0.0448	0.0045	1726.06	0.0004	0.0000	1799.35	0.0000	0.0000	1800.00
0.12	0.0694	0.0076	1685.47	0.0192	0.0021	1768.30	0.0001	0.0000	1799.82	0.0000	0.0000	1800.00
0.13	0.0325	0.0039	1746.31	0.0076	0.0009	1787.42	0.0000	0.0000	1799.95	0.0000	0.0000	1800.00
0.14	0.0141	0.0018	1776.71	0.0028	0.0004	1795.35	0.0000	0.0000	1799.99	0.0000	0.0000	1800.00
0.15	0.0057	0.0008	1790.60	0.0010	0.0001	1798.39	0.0000	0.0000	1800.00	0.0000	0.0000	1800.00
0.16	0.0022	0.0003	1796.45	0.0003	0.0000	1799.47	0.0000	0.0000	1800.00	0.0000	0.0000	1800.00
0.17	0.0008	0.0001	1798.74	0.0001	0.0000	1799.84	0.0000	0.0000	1800.00	0.0000	0.0000	1800.00
0.18	0.0003	0.0000	1799.58	0.0000	0.0000	1799.95	0.0000	0.0000	1800.00	0.0000	0.0000	1800.00
0.19	0.0001	0.0000	1799.87	0.0000	0.0000	1799.99	0.0000	0.0000	1800.00	0.0000	0.0000	1800.00
0.20	0.0000	0.0000	1799.96	0.0000	0.0000	1800.00	0.0000	0.0000	1800.00	0.0000	0.0000	1800.00

GRAPH 2

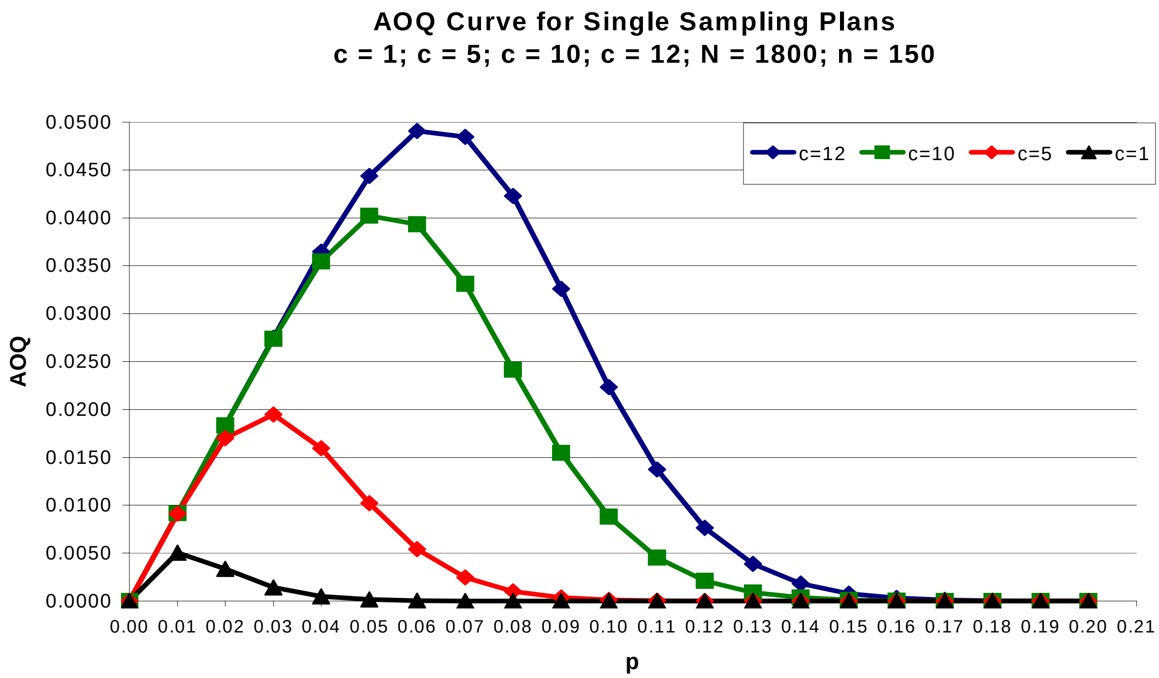
The OC Curve for Single Sampling Plans
c = 1; c = 5; c = 10; c = 12; N = 1800; n = 150



GRAPH 3



GRAPH 4

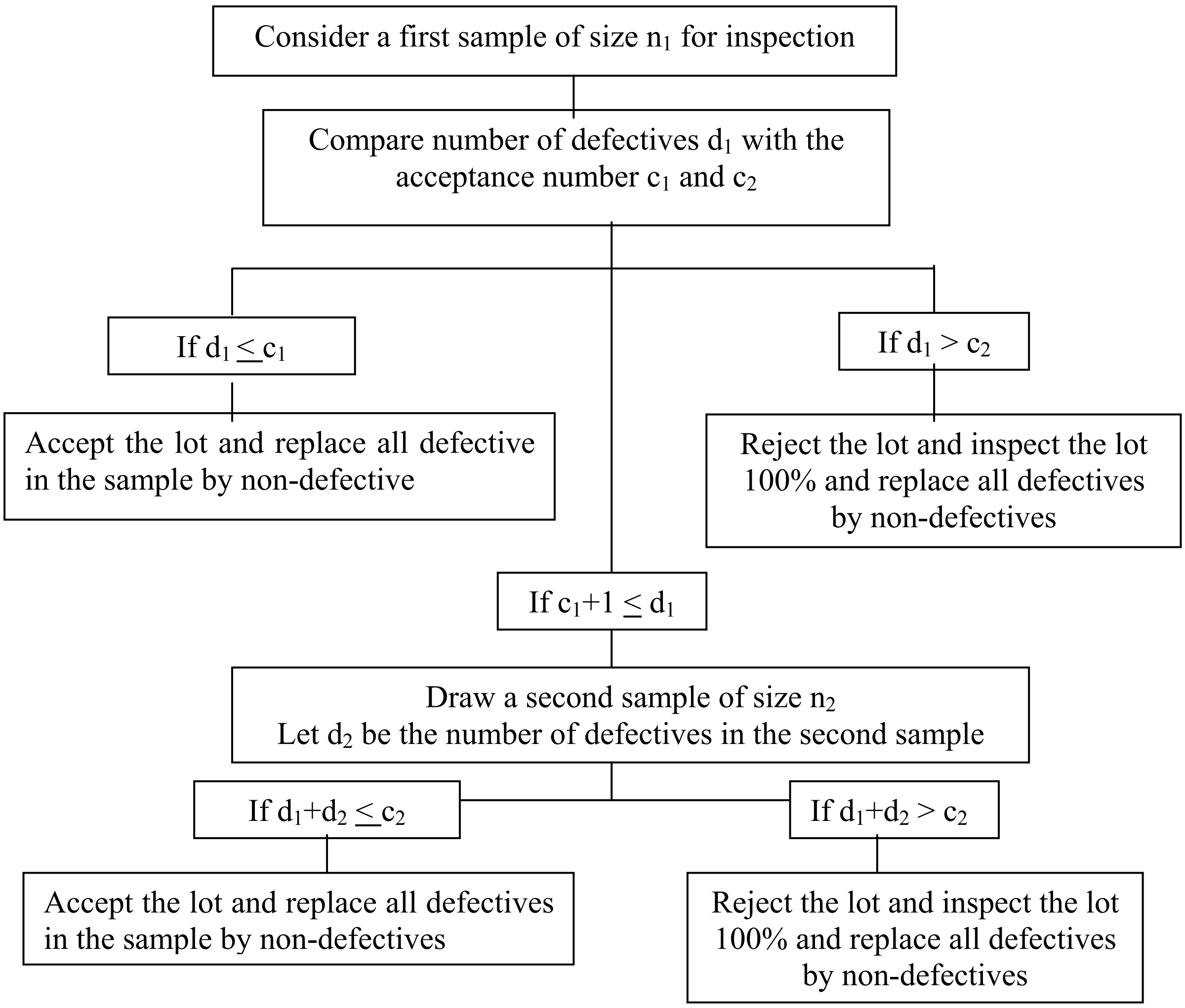


Rectifying Double Sampling Plan

Let

N = Lot size
n_1 = Size of the first sample
d_1 = Number of defectives observed in the first sample
c_1 = Acceptance number for the first sample
n_2 = Size of the second sample
d_2 = Number of defectives observed in the second sample
c_2 = Acceptance number for the second sample

Flow Diagram for Rectifying Double Sampling Inspection Plan



OC curve of Double Sampling Plan:

$$P_a(p)=\sum_{x=0}^{c_1}\binom{Np}{x}\binom{N-Np}{n_1-x}\Big/\binom{N}{n_1}$$
$$+\sum_{y=0}^{c_2-x}\sum_{x=c_1+1}^{c_2}\frac{\binom{Np}{x}\binom{N-Np}{n_1-x}\binom{Np-x}{y}\binom{N-n_1-(Np-x)}{n_2-y}}{\binom{N}{n_1}\binom{N-n_1}{n_2}} \qquad (11)$$
$$=P_{a1}(p)+P_{a2}(p)$$

where, x, denotes the number of defectives observed in the first sample and y denotes the number of defectives observed in the second sample, P_{a1} and P_{a2} are the probabilities of acceptance on the basis of first and second samples respectively.

The **Producer's risk** is given by:

$$P_a(\bar{p})=\sum_{x=0}^{c_1}\binom{N\bar{p}}{x}\binom{N-N\bar{p}}{n_1-x}\Big/\binom{N}{n_1}+\sum_{y=0}^{c_2-x}\sum_{x=c_1+1}^{c_2}\frac{\binom{N\bar{p}}{x}\binom{N-N\bar{p}}{n_1-x}\binom{N\bar{p}-x}{y}\binom{N-n_1-(N\bar{p}-x)}{n_2-y}}{\binom{N}{n_1}\binom{N-n_1}{n_2}} \quad ....(12)$$

and
The **Consumer's risk** is given by:

$$P_a(p_t)=\sum_{x=0}^{c_1}\binom{Np_t}{x}\binom{N-Np_t}{n_1-x}\Big/\binom{N}{n_1}+\sum_{y=0}^{c_2-x}\sum_{x=c_1+1}^{c_2}\frac{\binom{Np_t}{x}\binom{N-Np_t}{n_1-x}\binom{Np_t-x}{y}\binom{N-n_1-(Np_t-x)}{n_2-y}}{\binom{N}{n_1}\binom{N-n_1}{n_2}} \quad (13)$$

Also,

$$ATI = n_1P_{a1}+(n_1+n_2)(P_a-P_{a1})+N(1-P_a)$$
$$= n_1+n_2(1-P_{a1})+(N-n_1-n_2)(1-P_a) \qquad (14)$$

as only n_1 units will be inspected if the lot is accepted on the basis of the first sample or (n_1+n_2) units will be inspected if the lot is accepted on the basis of the second sample or all the N units will be inspected if the lot is rejected.
and

$$\text{AOQ} = p\left(\frac{N-n_1}{N}\right)\sum_{x=0}^{c_1}\binom{Np}{x}\binom{N-Np}{n_1-x}\Big/\binom{N}{n_1}$$
$$+p\left(\frac{N-n_1-n_2}{N}\right)\sum_{y=0}^{c_2-x}\sum_{x=c_1+1}^{c_2}\frac{\binom{Np}{x}\binom{N-Np}{n_1-x}\binom{Np-x}{y}\binom{N-n_1-(Np-x)}{n_2-y}}{\binom{N}{n_1}\binom{N-n_1}{n_2}} \qquad (15)$$

NOTE: In all the plans that we have discussed so far one major issue that emerges is the determination of n and c, as the value of the lot size N will always be given. Consumer's specifications will fix the values of p_t and P_c. Thus equation (8) becomes a function of two unknowns viz. n and c. Out of a large number of pairs of n and c satisfying (8), one should select that pair for which (9) is minimum (keeping in mind producer's interest also).

Another way of determining n and c is that consumer's interests will satisfy the value of AOQL, N in any case will be given to us. Thus expression (5) becomes a function of two unknowns viz. n and c. Again that pair of n and c satisfying (5) is selected for which (9) is minimum (taking care of producer's interest also).

EXAMPLE-2

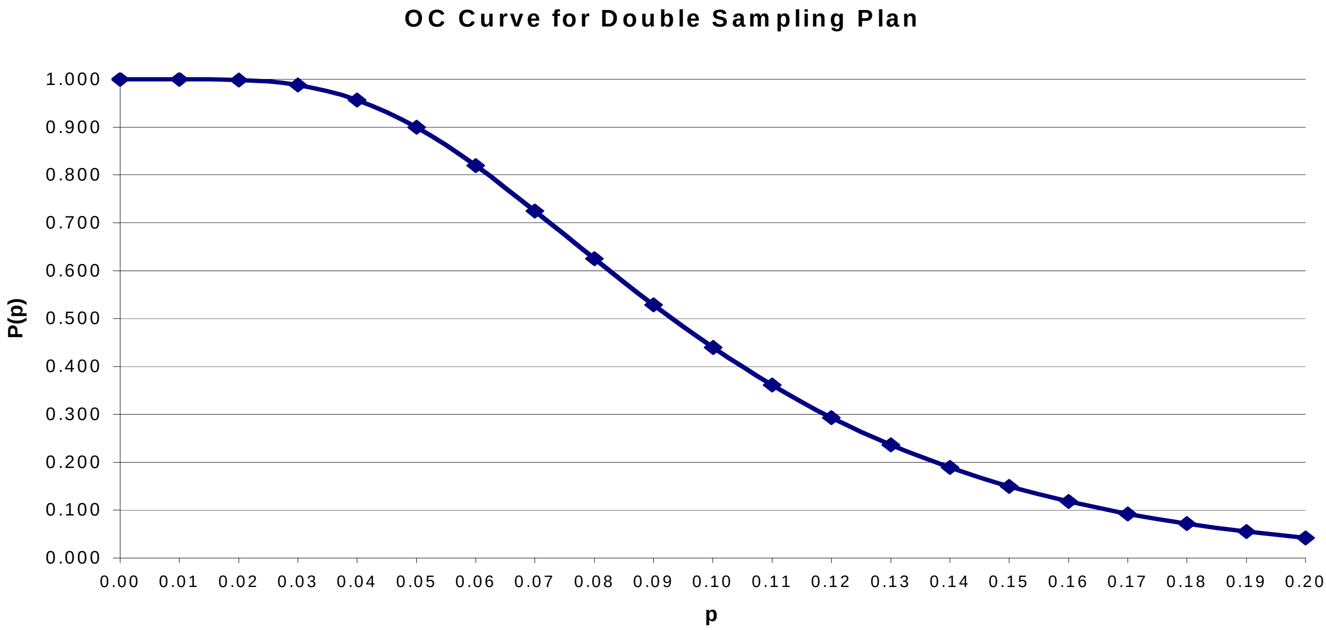
Draw the OC curve for a double sampling plan when N = 1000, n_1 = 30, n_2 = 50, c_1 = 2 and c_2 = 5. Also draw an ATI and AOQ curve for the above plan.

SOLUTION

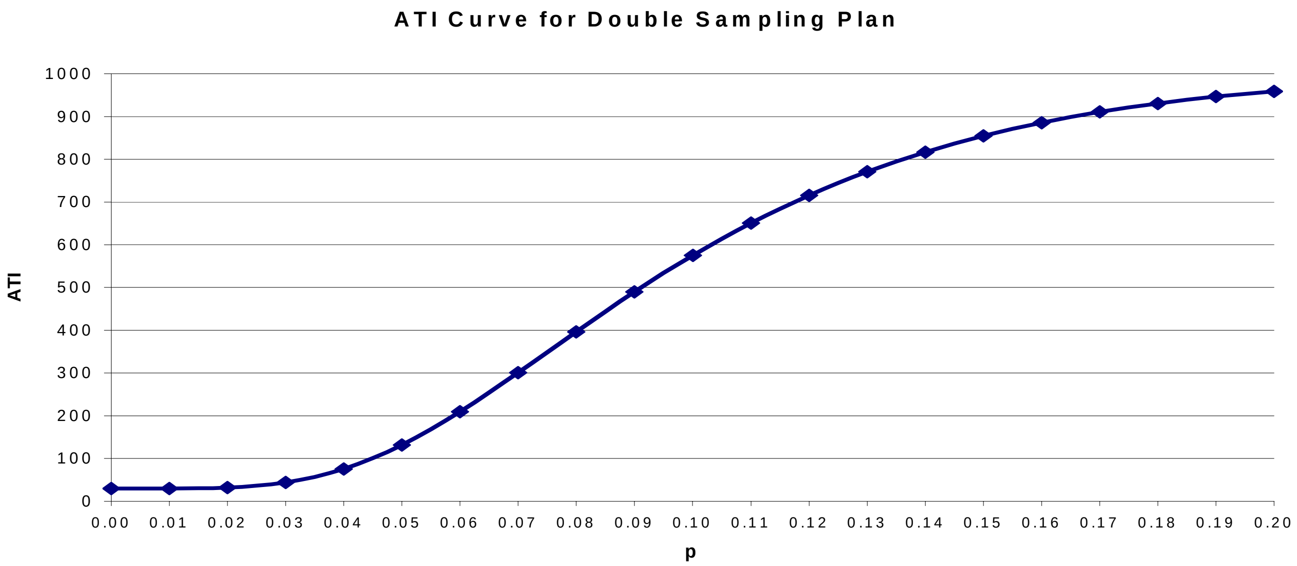
The following table gives the values of Pp(p), AOQ and ATI for double sampling plan:

p	$P_p(p)$	AOQ	ATI	p	$P_p(p)$	AOQ	ATI
0.01	0.999986	0.00970	30.140	0.11	0.361115	0.03842	650.756
0.02	0.998559	0.01935	32.310	0.12	0.293520	0.03409	715.935
0.03	0.987945	0.02868	43.955	0.13	0.236466	0.02977	771.018
0.04	0.956899	0.03699	75.358	0.14	0.188974	0.02563	816.920
0.05	0.899802	0.04343	131.465	0.15	0.149873	0.02179	854.749
0.06	0.819655	0.04745	209.242	0.16	0.117977	0.01830	885.630
0.07	0.724969	0.04896	300.597	0.17	0.092179	0.01519	910.622
0.08	0.625348	0.04828	396.477	0.18	0.071486	0.01248	930.677
0.09	0.528586	0.04594	489.544	0.19	0.055022	0.01014	946.637
0.10	0.439687	0.04249	575.082	0.20	0.042032	0.00815	959.233

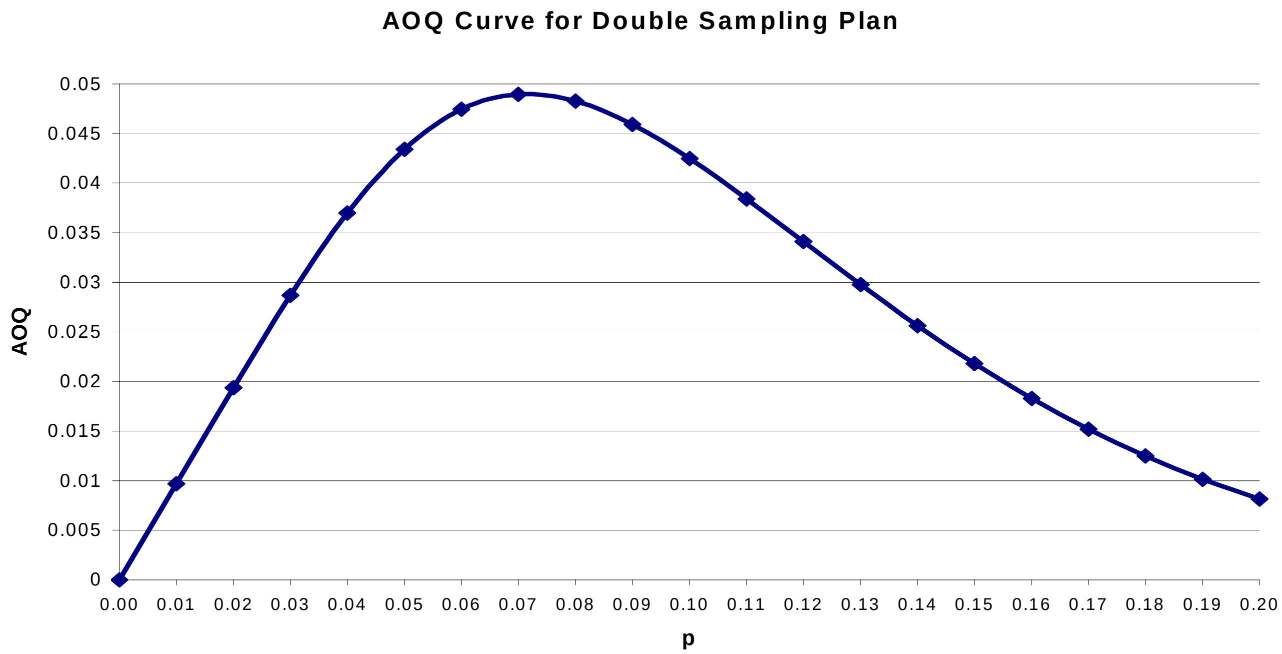
GRAPH 5



GRAPH 6



GRAPH 7



Sequential sampling plans

A sequential sampling plan invariably determines the number of units to be inspected.

Sequential sampling is based on the sequential probability ratio test (SPRT) developed by Wald (1947). Let p be the incoming lot quality. Also under H_0: $p = p_0$ and under H_1: $p = p_1$. Further, let x_i be the sample observation taken at the i^{th} stage (i = 1,….,m) then X_i is a bernoulli variate that is,

$X_i = 1$, if the i^{th} inspected item is defective

$= 0$, otherwise.

The sequential sampling plan is based on the likelihood ratio:

$$\frac{p_{1m}}{p_{0m}} = \frac{\prod_{i=1}^{m} p_1^{x_i}(1-p_1)^{1-x_i}}{\prod_{i=1}^{m} p_0^{x_i}(1-p_0)^{1-x_i}} \qquad (16)$$

Let d_m be the cumulative observed number of defectives found up to the m^{th} stage, then

$$\frac{p_{1m}}{p_{0m}} = \frac{p_1^{d_m}(1-p_1)^{m-d_m}}{p_0^{d_m}(1-p_0)^{m-d_m}} \qquad (17)$$

According to a sequential sampling plan the course of action to be followed at each stage of the experiment is as follows:

At the m^{th} stage of sampling which is reached only when no decision has been made beforehand:
Accept the lot if

$$\boxed{\frac{p_{1m}}{p_{0m}} \leq \frac{\beta}{1-\alpha}} \qquad (18)$$

Reject the lot if

$$\boxed{\frac{p_{1m}}{p_{0m}} \geq \frac{1-\beta}{\alpha}} \qquad (19)$$

and continue sampling by taking an additional observation if

$$\boxed{\frac{\beta}{1-\alpha} < \frac{p_{1m}}{p_{0m}} < \frac{1-\beta}{\alpha}} \qquad (20)$$

Taking log on both the side and simplifying, the above rule may be restated as:
Accept the lot if

$$\boxed{d_m \leq a_m} \qquad (21)$$

Reject the lot if

$$\boxed{d_m \geq r_m} \qquad (22)$$

and continue sampling by taking one more observation if

$$\boxed{a_m < d_m < r_m} \qquad (23)$$

where

$$\boxed{a_m = \frac{\log\frac{\beta}{1-\alpha}}{\log\frac{p_1(1-p_0)}{p_0(1-p_1)}} + m\frac{\log\frac{1-p_0}{1-p_1}}{\log\frac{p_1(1-p_0)}{p_0(1-p_1)}}} \qquad (24)$$

$$\boxed{r_m = \frac{\log\frac{1-\beta}{\alpha}}{\log\frac{p_1(1-p_0)}{p_0(1-p_1)}} + m\frac{\log\frac{1-p_0}{1-p_1}}{\log\frac{p_1(1-p_0)}{p_0(1-p_1)}}} \qquad (25)$$

Graphically a sequential sampling plan may be performed by plotting on the x-axis m, the total number of items inspected up to that time and on Y-axis d_m, the total number of observed defectives. We then draw the two lines viz. acceptant line $d_m = a_m$ and rejection line $d_m = r_m$. If the plotted points (m,d_m) lie within the boundaries of the acceptance and rejection lines, sampling is to be continued. Whenever a sample point fall on or below the lower line, the lot is accepted, whereas when ever a sample point falls on or above the upper line the lot is to be rejected.

OC Curve for Sequential Sampling Plan

For plotting OC curve for sequential sampling plan one need to consider minimum following five points:

p	**$P_p(p)$**
0	1
p_0	1-α
$\dfrac{\log\left(\dfrac{1-p_0}{1-p_1}\right)}{\log\left(\dfrac{p_1(1-p_0)}{p_0(1-p_1)}\right)}$	$\dfrac{\log\left(\dfrac{1-\beta}{\alpha}\right)}{\log\left(\dfrac{1-\alpha}{\beta}\right)+\log\left(\dfrac{1-\beta}{\alpha}\right)}$
p_1	β
1	0

ASN for Sequential Sampling Plan

Similarly, to plot ASN curve for sequential sampling plan minimum following five points are required to plot:

p	**ASN**
0	$\dfrac{\log\left(\dfrac{1-\alpha}{\beta}\right)}{\dfrac{\log\left(\dfrac{1-p_0}{1-p_1}\right)}{\log\left(\dfrac{p_1(1-p_0)}{p_0(1-p_1)}\right)}}$
p_0	$\dfrac{(1-\alpha)\log\left(\dfrac{1-\alpha}{\beta}\right)-\alpha\log\left(\dfrac{1-\beta}{\alpha}\right)}{\dfrac{\log\left(\dfrac{1-p_0}{1-p_1}\right)}{\log\left(\dfrac{p_1(1-p_0)}{p_0(1-p_1)}\right)}-p_0}$
$\dfrac{\log\left(\dfrac{1-p_0}{1-p_1}\right)}{\log\left(\dfrac{p_1(1-p_0)}{p_0(1-p_1)}\right)}$	$\dfrac{\log\left(\dfrac{1-\alpha}{\beta}\right)\log\left(\dfrac{1-\beta}{\alpha}\right)}{\dfrac{\log\left(\dfrac{1-p_0}{1-p_1}\right)}{\log\left(\dfrac{p_1(1-p_0)}{p_0(1-p_1)}\right)}\left(1-\dfrac{\log\left(\dfrac{1-p_0}{1-p_1}\right)}{\log\left(\dfrac{p_1(1-p_0)}{p_0(1-p_1)}\right)}\right)}$

p_1	$$\frac{(1-\beta)\log\left(\frac{1-\beta}{\alpha}\right)-\beta\log\left(\frac{1-\alpha}{\beta}\right)}{p_1-\frac{\log\left(\frac{1-p_0}{1-p_1}\right)}{\log\left(\frac{p_1(1-p_0)}{p_0(1-p_1)}\right)}}$$
1	$$\frac{\log\left(\frac{1-\beta}{\alpha}\right)}{1-\frac{\log\left(\frac{1-p_0}{1-p_1}\right)}{\log\left(\frac{p_1(1-p_0)}{p_0(1-p_1)}\right)}}$$

AOQ for sequential sampling plan:

$$\boxed{AOQ = P_a p} \tag{26}$$

and

ATI for sequential sampling plan:

$$\boxed{ATI = \frac{\log\left(\frac{1-\beta}{\alpha}\right)}{\log\left(\frac{1-\alpha}{\beta}\right)}\left(\frac{\log\frac{\beta}{1-\alpha}}{p\log\left(\frac{p_1}{p_0}\right)+(1-p)\log\left(\frac{1-p_1}{1-p_0}\right)}\right)+\left(1-\frac{\log\left(\frac{1-\beta}{\alpha}\right)}{\log\left(\frac{1-\alpha}{\beta}\right)}\right)N} \tag{27}$$

A sequential sampling plan has a number of advantages:

1 It reduces the amount of inspection to the minimum possible.

2 These plans do not involve any distribution problem. One has to simply work in terms of p_0, p_1, $1-\alpha$ and β.

EXAMPLE-3

For a sequential sampling procedure a sequence of defective and non-defective items selected from a lot is as follows:

“ N N N N N D N N N N N D N N N N N N N N N N N N N D N N

N N D N N N D”

Further, information provided to us in two different cases is as follows:

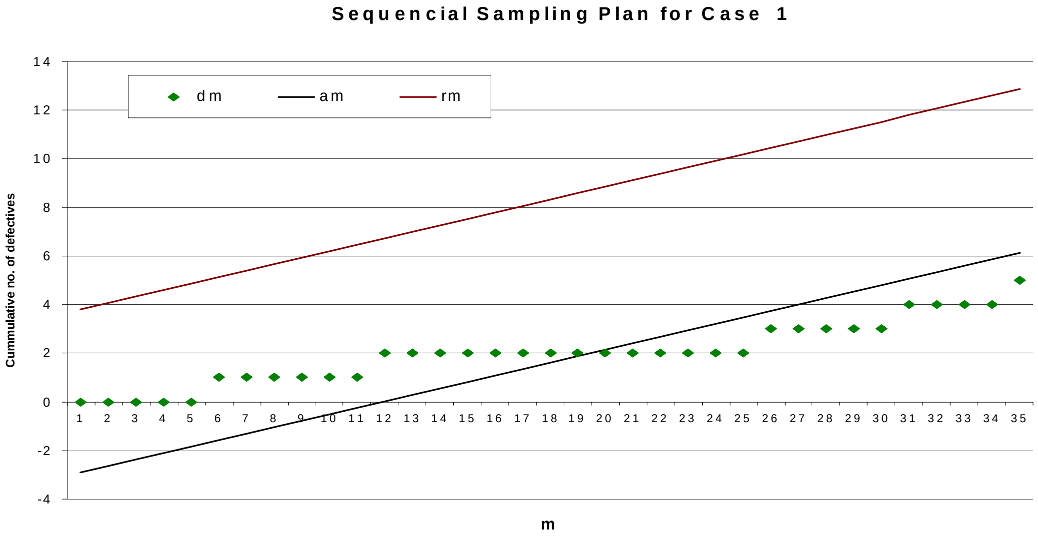
	Case 1	Case 2
α (Producer's Risk)	.02	.009
β (Consumer's Risk)	.03	.018
p_0 (AQL)	.17	.10
p_1 (LTFD)	.38	.37

Decision Line	Case 1	Case 2
Acceptance Line	$a_m = -3.1808 + 0.2661m$	$a_m = -2.4074 + 0.2142m$
Rejection Line	$r_m = 3.5413 + 0.2661m$	$r_m = 3.5413 + 0.2661m$

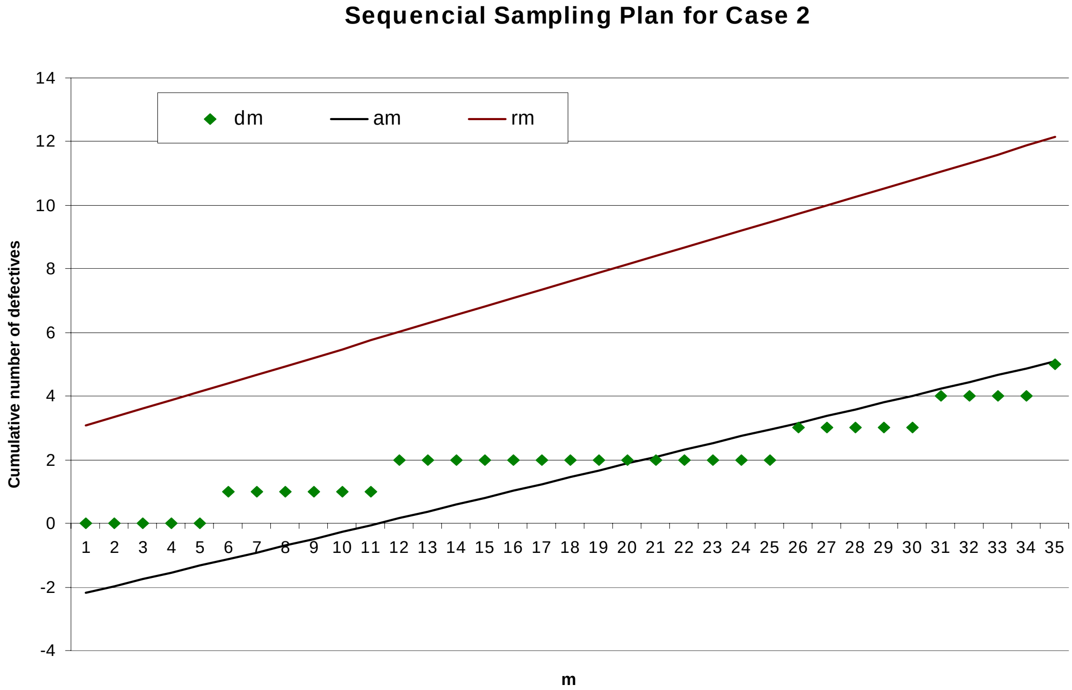
The sequential sampling plan and its graph for both the cases is as follows:

Case 1				Case 2			
m	d_m	a_m	r_m	m	d_m	a_m	r_m
1	0	-2.9146	3.8075	1	0	-2.1932	3.0844
2	0	-2.6485	4.0736	2	0	-1.9790	3.3505
3	0	-2.3823	4.3397	3	0	-1.7647	3.6166
4	0	-2.1162	4.6059	4	0	-1.5505	3.8828
5	0	-1.8501	4.8720	5	0	-1.3363	4.1489
6	1	-1.5839	5.1381	6	1	-1.1221	4.4150
7	1	-1.3178	5.4043	7	1	-0.9079	4.6812
8	1	-1.0517	5.6704	8	1	-0.6937	4.9473
9	1	-0.7855	5.9365	9	1	-0.4794	5.2135
10	1	-0.5194	6.2027	10	1	-0.2652	5.4796
11	1	-0.2533	6.4688	11	1	-0.0510	5.7457
12	2	0.0129	6.7350	12	2	0.1632	6.0119
13	2	0.2790	7.0011	13	2	0.3774	6.2780
14	2	0.5452	7.2672	14	2	0.5917	6.5441
15	2	0.8113	7.5334	15	2	0.8059	6.8103
16	2	1.0774	7.7995	16	2	1.0201	7.0764
17	2	1.3436	8.0656	17	2	1.2343	7.3425
18	2	1.6097	8.3318	18	2	1.4485	7.6087
19	2	1.8758	8.5979	19	2	1.6627	7.8748
20	2	2.1420	8.8640	20	2	1.8770	8.1409
21	2	2.4081	9.1302	21	2	2.0912	8.4071
22	2	2.6742	9.3963	22	2	2.3054	8.6732
23	2	2.9404	9.6624	23	2	2.5196	8.9394
24	2	3.2065	9.9286	24	2	2.7338	9.2055
25	2	3.4727	10.1947	25	2	2.9481	9.4716
26	3	3.7388	10.4609	26	3	3.1623	9.7378
27	3	4.0049	10.7270	27	3	3.3765	10.0039
28	3	4.2711	10.9931	28	3	3.5907	10.2700
29	3	4.5372	11.2593	29	3	3.8049	10.5362
30	3	4.8033	11.5254	30	3	4.0191	10.8023
31	4	5.0695	11.7915	31	4	4.2334	11.0684

		Case 1				Case 2	
m	d_m	a_m	r_m	m	d_m	a_m	r_m
32	4	5.3356	12.0577	32	4	4.4476	11.3346
33	4	5.6017	12.3238	33	4	4.6618	11.6007
34	4	5.8679	12.5899	34	4	4.8760	11.8669
35	5	6.1340	12.8561	35	5	5.0902	12.1330



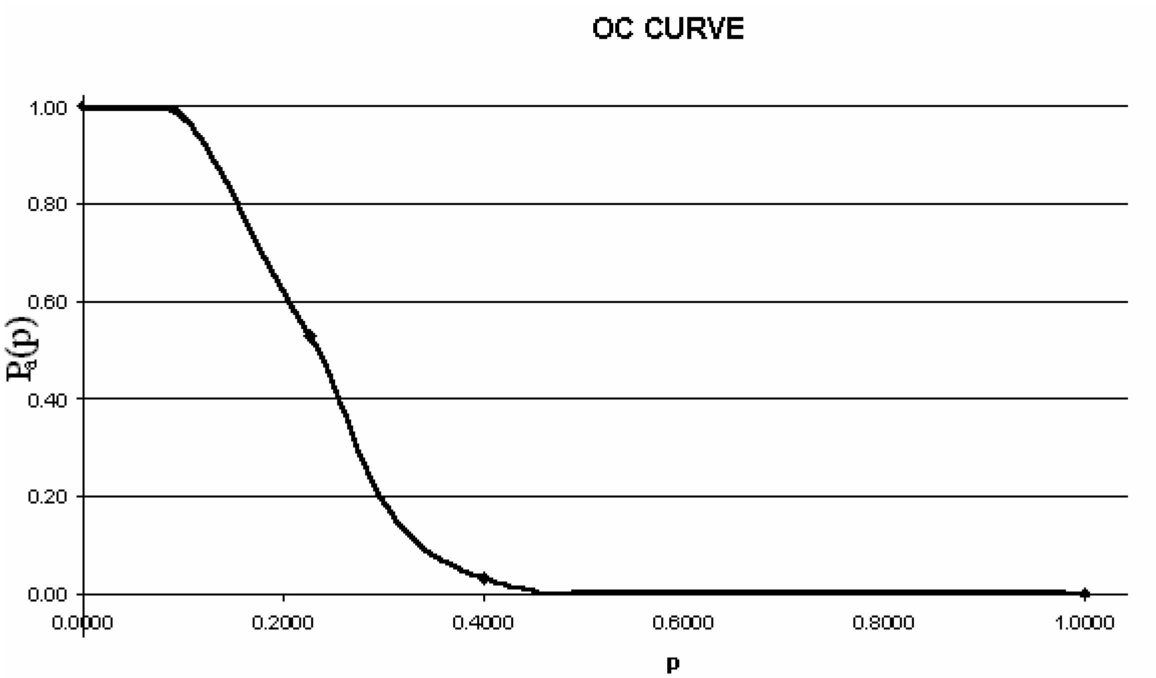
GRAPH 8



GRAPH 9

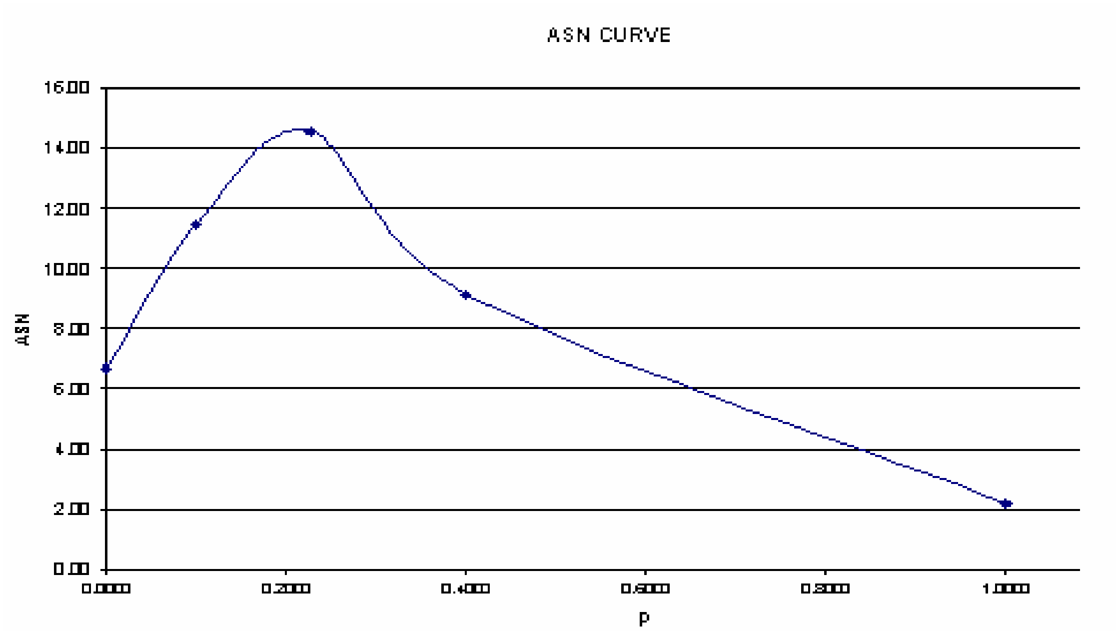
In case 1, at m = 20, dm < am, thus after inspecting the 20th item the sampling will be terminated. Whereas in case 2, dm < am for m = 21, thus sampling procedure will terminate after inspecting item number 21.

For the above example (Case 1) the OC-curve and the ASN curves are as follows



GRAPH 10

:



GRAPH 11

Acceptance sampling by variables

As opposed to acceptance sampling by attributes where we simply classify an item as defective or non-defective, in variable sampling plans measurements are taken on the quality characteristics of interest for each item. These plans are generally based on the sample average and sample standard deviation of the quality characteristics. A variable sampling plan has a number of advantages over attributes sampling plan. It requires smaller samples than attributes acceptance sampling plans for the same protection against various types of errors. Secondly, it provides more information about the manufacturing process. Thirdly, variable sampling plans become quite attractive when acceptable quality levels are very small because the sample sizes required by attributes sampling plans are very large.

The cost of inspection per item is higher in the measurement data than attributes data. However, the reduction in the sample size more than compensates the increased cost of inspection.

Variable sampling plans have a number of limitations. They require that the distribution of the quality characteristic under study must be known. Also, most of them assume it to be normal. However, in case it is not normal there may be serious risks of taking wrong actions. Further, we need to have a separate sampling plan for every variable that is being measured. For example, if an item is inspected for 8 variables, we need to have eight separate variable inspection sampling plans whereas, only one sampling plan is needed to take a decision about the lot for attributes sampling procedure.

Let y be the quality characteristic under study which is assumed to be normally distributed in the lot with parameters μ and σ respectively. Let USL be the upper specification limit and LSL be the lower specification limit then

1. If both USL & LSL are given then an item is considered as defective if $y < LSL$ or $y > USL$
2. If only USL is given then an item is considered as defective if $y > USL$
3. If only LSL is given then an item is considered as defective if $y < LSL$

Define the proportion defectives p_{USL} and p_{LSL} as:

$$\boxed{\begin{aligned} p_{USL} &= \frac{1}{\sigma\sqrt{2\pi}} \int_{USL}^{\infty} e^{[-(y-\mu)^2/2\sigma^2]} dy \\ &= 1-\varphi\left(\frac{USL-\mu}{\sigma}\right) \end{aligned}} \qquad (28)$$

and

$$\boxed{\begin{aligned} p_{LSL} &= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{LSL} e^{[-(y-\mu)^2/2\sigma^2]} dy \\ &= \varphi\left(\frac{LSL-\mu}{\sigma}\right) = \varphi\left[-\left(\frac{\mu-LSL}{\sigma}\right)\right] \end{aligned}} \qquad (29)$$

Now a lot will be considered as good or bad on the basis of the proportion defective. However, both p_{USL} and p_{LSL} are unknown quantities since μ and σ are unknown.

Nevertheless, their estimated values can be computed on the basis of sample mean $\bar{y}$ and sample s.d. s.

To take a decision about the lot we consider two cases:

1. **when σ is known and μ is unknown**: the minimum-variance unbiased estimators of p_{USL} and p_{LSL} are:

$$\boxed{\hat{p}_{USL}=1-\varphi\left(\sqrt{\frac{n}{n-1}}\left(\frac{USL-\bar{y}}{\sigma}\right)\right)} \tag{30}$$

and

$$\boxed{\hat{p}_{LSL}=\varphi\left[-\sqrt{\frac{n}{n-1}}\left(\frac{\bar{y}-LSL}{\sigma}\right)\right]} \tag{31}$$

thus, when both the specification limits are given, the lot is to be accepted

if $\hat{p}_{USL}+\hat{p}_{LSL}\le M$ (where M is the maximum allowable fraction defective) (32)
otherwise it is to be rejected.

Similarly, when upper specification limit is given, the lot is to be accepted

if $\boxed{\hat{p}_{USL}\le M \text{ or } \frac{USL-\bar{y}}{\sigma}\ge k}$, where k is a function of the standard normal variate (33)

When lower specification limit is given, the lot is to be accepted

if $\boxed{\hat{p}_{LSL}\le M \text{ or } \frac{\bar{y}-LSL}{\sigma}\ge k}$. (34)

2. **When both μ and σ are unknown:** the minimum-variance unbiased estimators of p_{USL} and p_{LSL} are the functions of $\frac{USL-\bar{y}}{s}$ and $\frac{\bar{y}-LSL}{s}$ respectively where $s=\sqrt{\left(\sum_{i=1}^{n}(y_i-\bar{y})^2\right)/(n-1)}$. In this case if both the specification limits are known the lot is accepted

   if $\boxed{\hat{p}_{USL}+\hat{p}_{LSL}\le M}$ (35)

Similarly, when upper specification limit is given, the lot is accepted

if $\boxed{\hat{p}_{USL}\le M \text{ or } \frac{USL-\bar{y}}{s}\ge k}$ (36)

and when lower specification limit is given, the lot is accepted

if $\boxed{\hat{p}_{LSL}\le M \text{ or } \frac{\bar{y}-LSL}{s}\ge k}$ (37)

PRACTICE PROBLEMS

Question 1

Draw OC, ASN and ATI curves for the following situations under single sampling plan.

N	10,000	10,000	10,000
n	275	400	500
c	9	12	15

and compare.

Question 2

Draw OC, ASN and ATI curves for the following situations under double sampling plan

	Case 1	**Case 2**	**Case 3**
N	4500	4500	4500
n1	110	112	115
n2	225	250	290
c1	3	3	4
c2	15	16	18

Question 3

For a sequential sampling procedure a sequence of defective and non-defective items selected from a lot is as follows:

" N N N N N D N N N N N D N N N N N N N N N N N N N D D N
N N N D N N N"

Further information provided to us in two different cases is as follows:

	Case 1	**Case 2**
α (Producer's Risk)	.02	.01
β (Consumer's Risk)	.03	.02
p_0 (AQL)	.10	.20
p_1 (LTFD)	.40	.45

Draw sequential sampling plans for both the cases and determine the number of items to be inspected before terminating the procedure for the given sequence of defective and non-defective items. Also draw OC and ASN curves for the above data.

References

1. Berenson, M.L. and Levine, D. M. (1992), Basic Business Statistics Concepts and Applications, Prentice Hall
2. Gupta S.C. and Kapoor V.K., Fundamental of Applied Statistics, Sultan Chand & Sons.
3. Goon, A.M. Gupta, M.K. & Dasgupta, B. (1986), Fundamentals of Statistics, Vol. 2, The World Press Private Limited, Calcutta.
4. John, P.W.M. (1990) Statistical Methods in Engineering and Quality Assurance, John Wiley & Sons Inc.
5. Montgomery D.C. (1991), Introduction to Statistical Quality Control, John Wiley & Sons