

INDUSTRIAL STATISTICS AND OPERATIONAL MANAGEMENT

7. Inventory Management

Dr. Ravi Mahendra Gor

Associate Dean

ICFAI Business School

ICFAI HUse,

Nr. GNFC INFO Tower

S. G. Road

Bodakdev

Ahmedabad-380054

Ph.: 079-26858632 (O); 079-26464029 (R); 09825323243 (M)

E-mail: ravigor@hotmail.com

Contents

[Introduction](#)

[Types of Inventory](#)

[How to Measure Inventory](#)

[Reasons for Holding Inventories](#)

[Objectives of Inventory Control](#)

[Costs Involved in Inventory Problems](#)

[Deterministic Continuous Review Models](#)

Economic Order Quantity (EOQ) Model With Constant Rate of Demand

EOQ Model With Constant Demand and Shortages Allowed

Inventory Model With Finite Replenishment Rate (Production Rate), Constant Demand, and No Shortages

[The Continuous Review Model: When To Order](#)

[Operation of Periodic Review System](#)

[Stochastic Single – Period Model](#)

[Review Exercise](#)

7.1 Introduction

The **inventory** may be defined as the physical stock of good, units or economic resources that are stored or reserved for smooth, efficient and effective functioning of business. Many companies have wide-ranging inventories, consisting of many small items such as paper pads, pencils, and paper clips, and fewer big items such as trucks, machines, and computers. A particular company's inventory is related to the business in which it is engaged. A tennis shop has an inventory of tennis rackets, shoes, and balls. A television manufacturer has parts, subassemblies, and finished TV sets in its inventory. A theater has an inventory of seats, a restaurant has an inventory of tables and chairs, and a public accounting firm has an inventory of accountants.

Without inventories customer would have to wait until their orders were filled from a source or were produced. In general, however, customer will not like to wait for long period of time. Another reason for maintaining inventory is the price fluctuation of some raw material, (may be seasonal), it would be profitable for a buyer to procure a sufficient quantity of raw material at lower price and use it whenever needed. Some researchers also argue that maintaining inventories on display attracts more customers resulting increase in sale and profits.

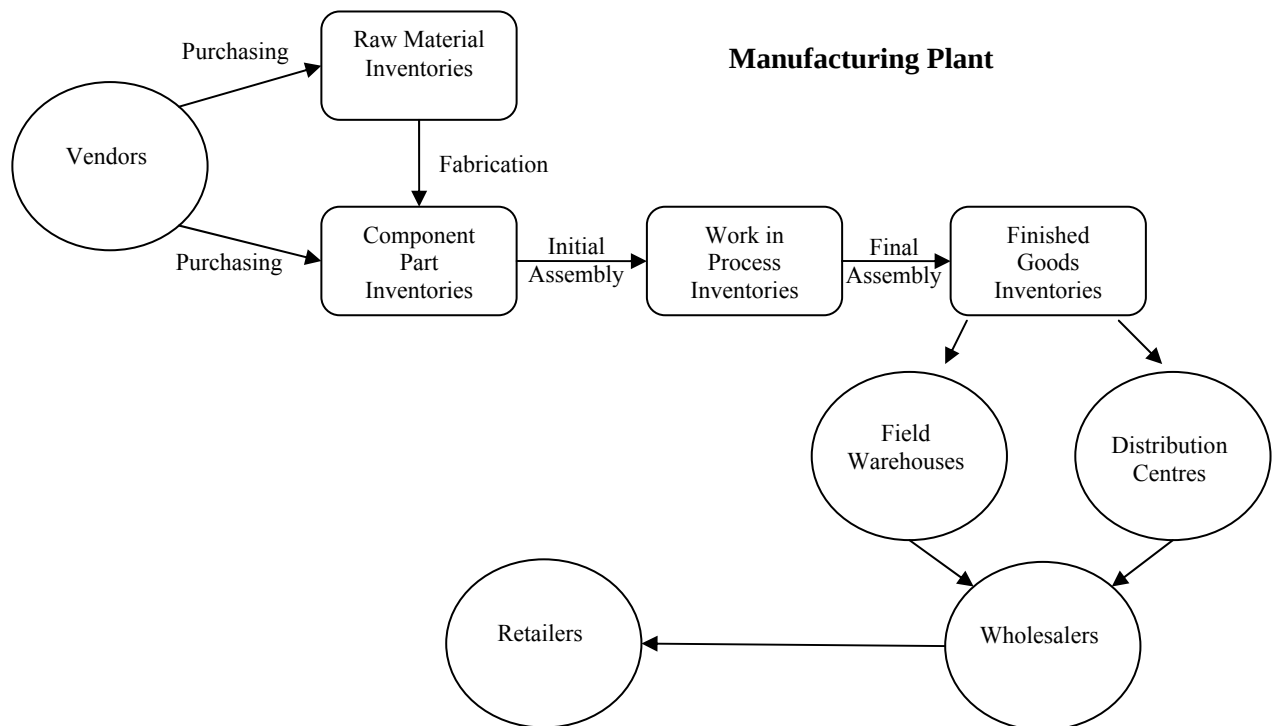
Just as inventory is the stock of any item or resource used in an organization, an **inventory system** or management is the set of policies and controls that monitor levels of inventory and determine what levels should be maintained, when stock should be replenished, and how large orders should be.

7.2 Types of Inventory:

The inventory is divided into two categories; viz direct inventory and indirect inventory. The direct inventory is one that is used for manufacturing the product. It is further sub-divided into following groups.

1. *Raw material inventories*
2. *Work-in-process inventories*
3. *Finished – goods inventory*
4. *Spare parts inventory*

As Figure 7.1 shows, a materials flow system has inventories in various forms. Inventories for a manufacturing facility consist of three major types, or accounting categories. Raw materials are the basic inputs to the manufacturing process. Work in process consists of partially finished goods. Finished goods are the outputs of the manufacturing process.



As Figure 7.1 shows, raw materials originate at the supplier and the manufacturing plant keeps them in inventory. The manufacturer then processes these raw materials into component parts, which are also inventories. The manufacturer may purchase other component parts directly from the supplier and then partly assemble those components, which creates a work-in-process inventory. Final assembly turns the work-in-process inventory into finish goods inventory. Manufacturers can hold their finished goods inventories at the plant, distribution centers, a field warehouse, wholesalers, and retailers.

Second Type of inventory is **indirect inventory**. The indirect inventory does not play any role in finished goods product but it is required for manufacturing. Thus, indirect inventory acts as catalyst which only speeds up / down the reaction. The indirect inventory is classified as follows:

1. **Fluctuation inventory** : This acts as an equilibrium between sales and production. The reserve stock that is kept to maintain fluctuations in the demand and lead – time, affecting the production of items is called fluctuation inventory.
2. **Anticipation inventory** : This is programmed in advance for the seasonal large sales, slack season, a plant shut down period etc.
3. **Transportation inventories** : The existence of transportation inventories is mainly due to movement of materials from one place to another.
4. **Decoupling Inventories** : These inventories are maintained for meeting out the demands during the decoupling period of manufacturing or purchasing.

7.3 How to Measure Inventory

Inventory is a hot topic in manufacturing circles today. Managers closely monitor and control inventories to keep them as low as possible while still providing acceptable customer service.

To monitor and control inventories, managers need ways to measure inventories. Typically inventories are measured in three ways: average aggregate inventory value, weeks of supply and inventory turnover.

Measuring inventories begins with a physical count of units, or a physical measurement of volume or weight. Because the monetary value of various units may vary a great deal, managers use the average aggregate inventory value to calculate the average total value of all items held in inventory during some time period. We compute the average aggregate inventory value by multiplying the average number of units of each item (the beginning inventory plus ending inventory, divided by two) by its per unit value to obtain the total average value of each item and then add the total average values of all items. The average aggregate inventory value tells the inventory manager just how much of the company's total assets are invested in inventory.

To calculate the second measure of inventory, weeks of supply, divide the average aggregate inventory value by the value of the sales per week. The numerator of this measure includes the value of all inventory items (raw materials, work in process, and finished goods), whereas the denominator includes only the cost of the finished goods sold.

To calculate the third measure of inventory, inventory turnover (turns), divide the annual value of the sales by the average aggregate inventory value maintained during the year.

7.4 Reasons for Holding Inventories

Inventories serve a number of important functions in various companies. Among the major reasons for holding inventories are

1. *To satisfy expected demand.* Companies use anticipation stock (buffer stock) to satisfy expected demand, and it is particularly important for products that exhibit marked seasonal demand but are produced at uniform rates. Air conditioner, rain suit manufacturers and children's toy manufacturers build up anticipation stock, which is depleted during peak demand periods.
2. *To protect against stock outs.* Manufacturers use safety stock to protect against uncertainties in either the demand or supply of an item. Delayed deliveries and unexpected increases in demand increase the risk of shortages. Safety stock provides insurance that the company can meet anticipated customer demand without backlogging orders. In Figure 7.1, the plant can invest in safety stocks at several points. Raw materials and component parts can have safety stocks within the manufacturing plant. Finished goods can have safety stocks throughout the materials flow (at the plant, field warehouses, distribution centers, wholesalers, and retailers).
3. *To take advantage of economic order cycles.* Companies use cycle stock to produce (or buy) in quantities larger than their immediate needs. Because of the cost involved in setting up a machine, companies usually find producing in large quantities economical. Similarly, to minimize purchasing costs companies often buy in quantities that exceed their immediate requirements. In both cases, periodic orders, or order cycles, produce more economical overall production costs. The quantity

produced is called the economic lot size. The quantity ordered is called the economic order quantity (EOQ).

4. *To maintain independence of operations.* The successive stages in the production and distribution system require a buffer of inventories between them so that they can maintain their independence of operations, for example, the raw materials inventory buffers the manufacturer from problems with a supplier. Similarly, the finished goods inventory buffers factory operations from problems in the distribution system.
5. *To allow for smooth and flexible production operations.* A production-distribution system needs flexibility and a smooth flow of material, but production cannot be instantaneous so work-in-process inventory relieve pressure on the production system. Similarly, manufacturers use in-transit or pipeline inventory to offset distribution delays. Both work-in-process inventories and pipeline inventories are part of a broader classification, called movement inventories.
6. *To guard against price increases.* Manufactures sometimes use large purchase, or large production runs, to achieve savings when they expect price increases for raw materials or component parts.

7.5 Objectives of inventory control

Inventory control has two major objectives. The first objective is to maximize the level of customer service by avoiding under stocking. Under stocking causes missed deliveries, backlogged orders, lost sales, production bottlenecks, and unhappy customers.

The second objective of inventory control is to promote efficiency in production or purchasing by minimizing the cost of providing adequate level of customer service. Placing too much emphasis on customer service can lead to over stocking, which means the company has tied up too much of its capital in inventories.

These two objectives often conflict. Achieving high levels of customer service by maintaining certain inventories leads to higher inventory costs and less efficiency in production or purchasing. Inventory control becomes a balancing act. Many a times a manager selects a desired level of customer service and attempts to control inventory in a manner that achieves that level of customer service at the lowest cost possible. Thus the problem is striking a balance in inventory levels, avoiding both overstocking and understocking.

The basic purpose of inventory analysis in manufacturing and stock keeping services is to specify

- (1) when items should be ordered (when to order)and
- (2) how large the order should be (how much to order).

Many firms are tending to enter into longer-term relationships with vendors to supply their needs for perhaps the entire year. This changes the "when" and "how many to order" to "when" and "how many to deliver."

In this chapter, we will try to answer these questions under variety of circumstances.

In making decisions about inventory levels, companies must address a variety of costs. Hence, before we proceed to answer the above two questions, let us discuss about the costs involved in inventory decisions.

7.6 Costs Involved In Inventory Problems :

The costs play an important role in making a decision to maintain the inventory in the organization. These costs are as follow :

1. Purchase Cost :

The actual price paid for the procurement of items is called ***purchase cost***. The ***purchase cost*** becomes relevant if a quantity discount is available. A company offering quantity discounts drops the price per item when the order is sufficiently large. This becomes an incentive to order greater quantities.

For us, in this chapter we will assume that it is independent of the size of the quantity ordered (or manufactured) or in other words, quantity discounts are not available.

2. Inventory Holding Cost :

The ***holding, carrying or storage cost*** is the cost associated with maintaining an inventory until it is used or sold. Holding or storage cost includes the cost of maintaining storage facilities, the cost of insuring the inventory, taxes attributed to storage, costs associated with obsolescence, and costs associated with the capital that is committed to the inventory. The latter is called the opportunity cost, an expense incurred by having capital tied up in inventory rather than having it invested elsewhere, and it is frequently the most important component of the inventory holding cost. The opportunity cost is generally equal to the biggest return that the company could obtain from alternative investments. The holding or storage cost is usually related to the maximum quantity, average quantity, or excess of supply in relation to demand during a particular time period. For example, a company could estimate that its annual inventory holding cost is approximately 13 to 15 percent of the original purchase price of the commodity. So another common practice is to estimate the annual holding costs as a percentage of the unit cost of the item.

3. Shortage Cost :

The ***stock out or shortage cost*** occurs when the demand for an item exceeds its supply. When a stock out or shortage occurs, a company faces two possibilities:

- It can meet the shortage with some type of rush, special handling or priority shipment.
- It cannot meet the shortage at all.

The cost associated with a stock out or shortage depends on how the company handles the problem. Consider first the cost incurred when the inventory is on back order. In theory, demand for the back-ordered item is satisfied when the item next becomes available. From a practical standpoint, accurately determining the nature and magnitude of the back-ordering cost can be difficult. A small portion of the back-ordering cost, such as the cost of notifying the customer that the item has been back ordered and when delivery can be expected, may be fairly easy to determine. Another portion of the back-ordering cost may involve explicit costs for overtime, special clerical and administrative costs for expediting, and extraordinary transportation charges. Such costs are much more difficult to determine. Finally, a major portion of the back-ordering cost is an implicit cost - it reflects the loss of the customer's goodwill. This is a difficult cost to measure, because it is a penalty cost that accounts for lost future sales, for example, an equipment retailer might have a shortage cost composed primarily of loss goodwill, which it can

estimate as 15 percent of the original purchase cost of the equipment. But when back ordering is not possible or the customer chooses to order from another company, the shortage costs include the costs of notifying the customer, the loss in profit, from the sale, and the future loss of goodwill.

The shortage cost may also depend on the size of the shortage and how long the shortage lasts. For example, customers may have written into their purchase contracts specific penalty clauses that are based on shortage amounts and times. In other instances the shortage cost may be a fixed amount regardless of the number of units unavailable or how long the shortage exists.

4. Set-Up Cost (or the ordering cost):

Each time a company places a purchase order with a supplier or a production order with its own shop, it incurs an **ordering cost**. To buy an item someone has to solicit and evaluate bids, negotiate price terms, decide how much to order, prepare purchase orders, and follow up to make sure that the shipment arrives on time. For example, when we order an item from a supplier, we might incur a Rs.100 cost for placing the order and a cost of Rs.5 for each unit we are ordering. The purchasing cost function for this situation is $\text{Rs.}100 + \text{Rs.}5x$ where x is the number of units. The fixed portion (Rs.100) of the total purchasing cost is independent of the amount ordered; it is primarily the cost of the clerical and administrative work. Placing a production order for a manufacturing item involves many of the same activities; only the type of paperwork changes.

The **setup cost** is the cost involved in changing a machine over to produce a different part or item. While someone is adjusting a machine, it is idle and the company incurs the additional costs of the setup workers. Sometimes the machines are producing trial products, and they will make defective parts until the machine is fine-tuned. For example, we have a production process for manufacturing TV cabinets. The setup cost for a production run is Rs.1,000, and each TV cabinet costs Rs. 55 to manufacture. The manufacturing cost function for this situation is $\text{Rs.}1,000 + 55x$ where x is the number of TV cabinets. Companies treat setup costs as a fixed cost and try to make the production lot size as big as possible to spread the setup cost over as many units as possible.

5. Total Inventory Cost :

If price discounts are available, then we should formulate total inventory cost by taking sum of purchase cost (PC), Inventory Holding Cost (IHC), Shortage Cost (SC) and Ordering Cost (OC). Thus, the total inventory cost; TC, is given by

$$\text{TC} = \text{PC} + \text{IHC} + \text{SC} + \text{OC}$$

When price discounts are not offered then total cost (TC) is given by

$$\text{TC} = \text{IHC} + \text{SC} + \text{OC}$$

Establishing the correct quantity to order from vendors or the size of lots submitted to the firm's productive facilities involves a search for the minimum total cost resulting from the combined effects of four individual costs: holding costs, setup costs, ordering costs, and shortage costs. Of course, the timing of these orders is a critical factor that may impact inventory cost.

Apart from costs, the other variables which play an important role in decision making are as follows:

1. Demand :

The size of the demand is the number of units required in each period. It is not necessarily the amount sold because demand may remain unfulfilled due to shortage or delays. The demand pattern of the items may be either deterministic or probabilistic. In the deterministic case, the demand over a period is known. This known demand may be fixed or variable with time. Such demand is known as **static** and **dynamic** respectively. When the demand over a period is uncertain but can be predicted by a probability distribution, we say it is a case of the probabilistic demand. A probabilistic demand may be stationary or non-stationary over time.

Independent versus Dependent Demand

In inventory management, it is important to understand the difference between dependent and independent demand. The reason is that entire inventory systems are predicated on whether demand is derived from an end item or is related to the item itself.

Briefly, the distinction between independent and dependent demand is this: In independent demand, the demands for various items are unrelated to each other. For example, a workstation may produce many parts that are unrelated but meet some external demand requirement. In dependent demand, the need for any one item is a direct result of the need for some other item, usually a higher-level item of which it is part.

In concept, dependent demand is a relatively straightforward computational problem. Needed quantities of a dependent-demand item are simply computed, based on the number needed in each higher-level item in which it is used. For example, if an automobile company plans on producing 500 cars per day, then obviously it will need 2,000 wheels and tires (plus spares). The number of wheels and tires needed is dependent on the production levels and is not derived separately. The demand for cars, on the other hand, is independent—it comes from many sources external to the automobile firm and is not a part of other products; it is unrelated to the demand for other products.

To determine the quantities of independent items that must be produced, firms usually turn to their sales and market research departments. They use a variety of techniques, including customer surveys, forecasting techniques, and economic and sociological trends as we discussed in the previous Chapter 6 on forecasting techniques. Because independent demand is uncertain, extra units must be carried in inventory. This chapter presents models to determine how many units need to be ordered, and how many extra units should be carried to reduce the risk of stocking out.

2. Lead – Time :

It is the time between placing an order and its realization in stock. It can be deterministic or probabilistic. If both demand and lead-time are deterministic, one needs to order in advance by a time equal to lead-time. However, if lead-time is probabilistic, it is very difficult to answer - when to order?

3. Cycle Time :

The cycle time is time between placements of two orders. It is denoted by T. It can be determined in one of the two ways.

- I) ***Continuous Review*** : Here the number of units of an item on hand is known. In this case, an order of fixed size is placed every time the inventory level reaches at a pre-specified

level, called **reorder level**. Many authors referred this as the **two-bin systems** or **fixed order level system**.

- II) **Periodic Review** : Here the orders are placed at equal interval of time and size of the order depends on the inventory on hand and on order at the time of the review. This is also called the **fixed order interval system**

4. Planning Horizon :

The period over which a particular inventory level will be maintained is called **planning horizon**. It may be finite or infinite. It is denoted by H.

7.7 Deterministic Continuous Review Models :

Notations :

We shall use the following notations for the discussion of models in the chapter.

C : Purchase or manufacturing cost of an item.
C_o: The ordering cost per order.
C_h : Inventory holding cost of an item per unit per time unit.
C_s : Shortage cost per unit short per time.
D : Demand rate of an item.
Q : Order quantity (a decision variable).
T : Cycle time (a decision variable).
ROL : Reorder level i.e. the level of inventory at which an order is placed.
L : Lead – Time.
N : Number of orders per time unit.
T : Production time period.
P : Production rate at which quantity is produced.
PC : Total purchase cost.
IHC : Total inventory holding cost per time unit
SC : Total shortage cost per time unit
OC : Ordering cost per order.
TC : Total inventory cost per time unit.
TVC : Total variable inventory cost per time unit.

The parameters are defined with proper units.

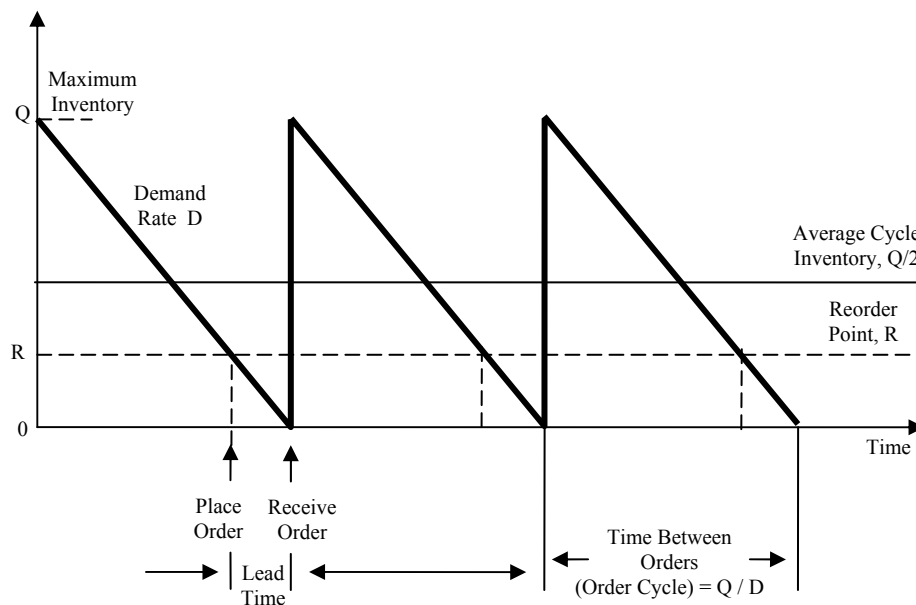
7.7.1 Economic Order Quantity (EOQ) Model With Constant Rate Of Demand :

EOQ is one of the oldest and most commonly known techniques. This model was first developed by Ford Harris and R.Wilson independently in 1915. The objective is to determine economic order quantity, Q which minimizes the total cost of an inventory system when demand occurs at a constant rate. The model is developed under following assumptions:

- ➔ The system deals with single item.
- ➔ The demand rate of D units per time unit is known and constant.
- ➔ Quantity discounts are not available.
- ➔ The item is produced in lots or purchasers are made in orders. The ordering cost is constant.
- ➔ Shortages are not allowed. Lead – Time is known and is constant.
- ➔ Replenishment rate is infinite.
- ➔ Replenishment size, Q , is the decision variable.
- ➔ T is cycle time.
- ➔ The inventory holding cost, C_h per unit per time unit is known and constant during the period under review.

The following figure shows the graphic depiction of this particular inventory situation. Each inventory cycle begins with the receipt of an order of Q units. i.e Q units are ordered and stocked in the system. Demand is occurring at the rate of D units per time unit during cycle time T .

At the reorder point R , when the on-hand inventory is barely sufficient to satisfy demand during the lead time, LT , an order of Q units is placed. Since the demand rate and the lead time are constant, the order of Q units is received exactly when the inventory level reaches zero. This means that there are no shortages.



The inventory level varies from Q to zero, so the average inventory level during the inventory cycle is $Q/2$. So, the inventory holding cost is obtained by multiplying this quantity with the cost of holding one unit per time unit. Hence, $IHC = (Q/2) C_h$. This cost is a linear function of Q .

The number of orders placed during the planning horizon would be D/Q and hence the inventory ordering cost OC will be a function of the number of orders placed and the ordering cost per order.

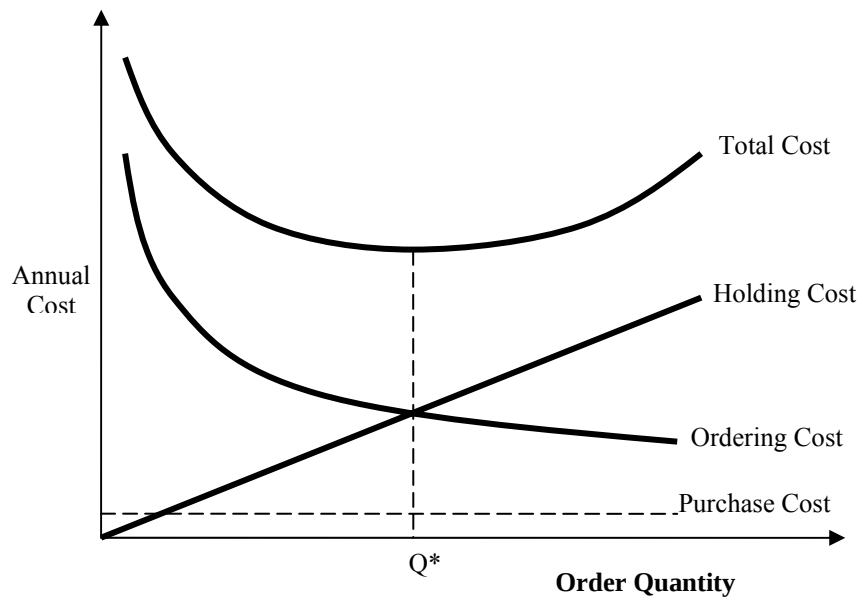
Thus, $OC = (D/Q) C_o$. Because the number of orders made in the planning horizon, D/Q , decreases as the order size, Q , increases, OC is inversely proportional to Q .

The cost of the individual item is assumed to be constant, regardless of the size of the order. So the purchase cost of the item is a horizontal line as shown in the following figure. It only increases the total inventory cost by the constant amount, DC , during the entire quantity range. It does not affect the optimal order quantity, Q^* . Therefore, it is not really a relevant cost for the economic order quantity decision and we can eliminate it from further consideration in the model.

Hence,

Total Inventory Cost $TC = \text{Ordering cost} + \text{holding cost}$

$$TC = (D/Q) C_o + (Q/2) C_h$$



The total cost curve is U-shaped and reaches its minimum at the quantity for which the carrying and the ordering costs are equal. We can equate both these values to obtain the optimal order quantity Q^* .

Alternatively, we can use calculus to obtain the expression for Q^* , setting the first derivative of TC to zero and solving for Q .

Thus,

$$TC = \left(\frac{D}{Q}\right)C_o + \left(\frac{Q}{2}\right)C_h$$

$$\frac{dTC}{dQ} = \left(-\frac{D}{Q^2}\right)C_o + \frac{C_h}{2} = 0$$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}}$$

Also, since $(d^2TC/dQ^2) > 0$, Q^* is minimum.

The resulting expression of Q^* obtained above is called the economic order quantity or the economic lot size.

The number of orders during the planning horizon is $= D/Q^*$

The length of the order cycle t is $= Q^*/D$

And the minimum total inventory cost $TC^* = (D/Q^*) C_o + (Q^*/2) C_h = \sqrt{2DC_o C_h}$

Note:

(1) Over a period of time, a company can use two policies for making inventory decisions. First, it can keep the order size small. This will result in a small average inventory, and inventory carrying costs will be low. But this policy will lead to frequent orders, and the total ordering costs will increase. Second, the company can increase its order size. This will result in less frequent orders, so the total ordering costs will be low. But this will result in a high average inventory, and the total inventory carrying costs will increase.

(2) If unit cost is taken into account then $TC = CD + (D/Q) C_o + (Q/2) C_h$

Sensitivity of lot-size model :

For the lot-size model, we have total cost of an inventory system per time unit as $TC = (D/Q^*) C_o + (Q^*/2) C_h$ and Q^* as given above. Now suppose that instead of ordering for Q_0 – units (given as Q^* above) and suppose that for it the total cost of the inventory system is $TC(Q_0)$, we replenish another lot-size (say) Q^1 . Such that $Q^1 = bQ_0$,

$b > 0$ and let $TC^1(Q^1)$ be the corresponding total cost of an inventory system.

The ratio $\frac{TC^1(Q^1)}{TC(Q_0)} = \frac{1+b^2}{2b}$ is known as **the measure of sensitivity** of the lot-size model.

Limitations Of The EOQ Formula :

Note that the EOQ formula is derived under several rigid assumptions which give rise to limitation on its applicability.

- ➔ In practice, the demand is neither known with certainty nor is uniform over the time period. If the fluctuations are mild, the formula is practically valid; but when fluctuations are wild, the formula loses its validity.
- ➔ It is not easy to measure the inventory holding cost and the ordering cost accurately. The ordering cost may not be fixed but will depend on the order quantity Q .
- ➔ The assumptions of zero lead-time and that the inventory level will reach to zero at the time of the next replenishment is not possible.
- ➔ The stock depletion is rarely uniform and gradual.
- ➔ One may have to take into account the constraints of floor-space, capital investment, etc in stocking the items in the inventory system.

Example 7.1 : Using the following information, obtain the EOQ and the total variable cost associated with the policy of ordering quantities of that size. Annual demand = 20,000 units, ordering cost = Rs. 150 per order, and inventory carrying cost is 24% of average inventory value.

Solution : Given $D = 20,000$ units, $C_o = \text{Rs. } 150$ / order, $C_h = \text{Rs. } 0.24$ / unit / annum. Then using the above formulas for Q and TC ,

$$Q^* = 5000 \text{ units and } TC(Q^*) = \text{Rs. } 1,200.$$

Example 7.2 : An oil engine manufacturer purchases lubricants at the rate of Rs. 42 per piece from a vendor. The requirement of these lubricants is 1,800 per year. What should be the order quantity per order, if the cost per placement of an order is Rs.16 and inventory carrying charge per rupee per year is 20 paise.

Solution : Given $D = 1,800 \times 42 = 75,600$ units, $C_o = \text{Rs. } 16$ / order and $C_h = 0.20$ per unit / year. Then

$$Q^* = 34,776 \text{ units.}$$

Thus, the optimum inventory quantity of lubricant at the rate of Rs.42 per lubricant $= Q^*/42 = 83$ lubricants.

Example 7.3 : A company uses rivets at a rate of 5,000 kg per year, rivets costing Rs.2 per kg. It costs Rs.20 to place an order and the carrying cost of inventory is 10 % per annum. How frequently should order for rivets be placed and how much?

Solution : Given $D = 5,000$ kg / year. $C = \text{Rs. } 2$ / kg., $C_o = \text{Rs. } 20$ / order and $C_h = 2 \times 10\%$ per unit/year.

Then $Q^* = 1,000$ kgs. and $T^* = Q^*/D = 1/5$ years = 2.4 months.

Example 7.4 : A supplier ships 100 units of a product every Monday. The purchase cost of product is Rs.60 per unit. The cost of ordering and transportation from the supplier is Rs.150 per order. The cost of carrying inventory is estimated at 15 % per year of the purchase cost. Find the lot-size that will minimize the cost of the system. Also determine the optimum cost.

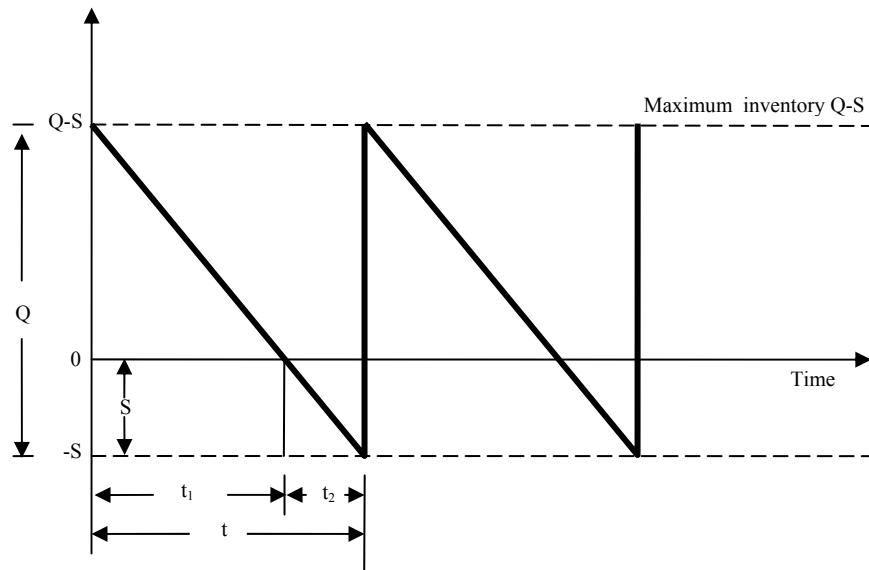
Solution : Given $D = 100$ units per week, $C_o = \text{Rs. } 150$ /order, $C_h = 15\%$ of $60 = \text{Rs. } 9$ per unit / year = $\text{Rs. } 9/52$ per unit / week. Then $Q^* = 416$ units, and optimum cost $= CD + TC(Q^*) = \text{Rs. } 6,072$

7.7.2 EOQ Model with constant demand and shortages allowed:

The inventory problem in the above section becomes slightly more complicated when a company permits shortages, or backorders, to occur. However, in many situations shortages are economically desirable. Permitting shortages allows the manufacturer or retailer to increase the cycle time, thereby spreading the setup or ordering cost over a longer time period. Allowing shortages may also be desirable when the unit value of the inventory and therefore the inventory holding cost is high.

In the back order situation customers place an order, no stock is available, and they simply wait until stock becomes available, at which point the order is filled. The company hopes that the waiting period for the back order will be short and its customers will be patient.

For this model we will use the assumptions of a known and constant demand rate and instantaneous delivery of goods to inventory like the basic EOQ model. If S represents the amount of the shortage (size of the back order) that has accumulated when the new shipment of size Q arrives, the economic order quantity model with constant demand and permissible shortages has the following major characteristics and graphic depiction.



- When the new shipment of size Q arrives, the company immediately ships the back orders of size S to the customers. The remaining units $Q-S$ immediately go into inventory.
- The inventory level will vary from a minimum of $-S$ units to a maximum of $Q-S$ units.
- The inventory cycle of T units is divided into two distinct parts: t_1 when inventory is available for filling orders and t_2 when inventory is not available, stock outs occur, and back orders are made.

Here apart from the notations introduced in the previous model we introduce two new notations as follows:

C_s : cost of back order, per unit per unit time

S : the number of units short or back ordered.

Now,

The inventory ordering cost is a function of the number of orders made, D/Q , and the inventory ordering cost per order, C_o .

$$OC = \text{No. of orders} \times \text{Cost per order} = (D/Q)C_o$$

Also we know that $t_1 = (Q-S)/D$ and $t_2 = S/D$

The inventory holding cost can be calculated from the figure as:

The average inventory for the time period $t = [(\text{Avg. inventory over } t_1) + (\text{Avg. inventory over } t_2)] / t$

The positive inventory level ranges from $Q-S$ to 0 . This means that the average inventory level is $(Q-S)/2$ for the time period t_1 . For t_2 it is 0 .

$IHC = \text{Average inventory} \times \text{cost of holding one unit}$

$$= \left(\frac{\frac{Q-S}{2}t_1 + 0t_2}{t} \right) C_h = \frac{(Q-S)^2}{2Q} C_h$$

The backordering cost is computed in a similar way. From the figure we can see that the shortage ranges from 0 units to S units. This means that the average shortage is S/2 while there are shortages i.e during the time period t_2 .

SC = Average number of units short $\times$ cost of one unit being short

$$= \frac{S}{2} t_2 C_s = \frac{S^2}{2Q} C_s$$

For this inventory model the total cost is calculated as

TC = ordering cost + holding cost + shortage cost = OC + IHC + SC

$$= \frac{D}{Q} C_o + \frac{(Q-S)^2}{2Q} C_h + \frac{S^2}{2Q} C_s$$

Since TC is a function of two variables Q and S, therefore to determine the optimal order size and the optimal shortage level S, we need to differentiate the total variable cost function with respect to Q and S, set the two resulting equations equal to zero and solve them simultaneously. By doing so, we get the following results

$$Q^* = \sqrt{\frac{2DC_o}{C_h} \left(\frac{C_s + C_h}{C_s} \right)}$$

$$S^* = Q^* \left(\frac{C_h}{C_s + C_h} \right)$$

The number of orders for the planning horizon = D/Q^*

The maximum inventory level = $Q^* - S^*$

The average positive inventory level is = $(Q^* - S^*) / Q^*$

The length of time during which there are no shortages = $t_1^* = (Q^* - S^*) / D$

The length of time during which there are shortages = $t_2^* = S^* / D$

The length of the inventory cycle = $t^* = Q^* / D$

The minimum total inventory cost during the planning horizon = $TC^* = \frac{D}{Q^*} C_o + \frac{(Q^* - S^*)^2}{2Q^*} C_h + \frac{S^{*2}}{2Q^*} C_s$

When values of Q^* and S^* are substituted, we get $TC^* = \sqrt{2DC_o C_h \left(\frac{C_s}{C_s + C_h} \right)}$

Example 7.5 : The demand for a certain item is 16 units per period. Unsatisfied demand causes a shortage cost of Rs.0.75 per unit per short period. The cost of initiating purchasing action is Rs. 15.00 per purchase and the holding cost is 15% of average inventory valuation per period. Item cost is Rs. 8.00 per unit. Find the minimum cost and purchase quantity.

Solution : Given $D = 16$ units, $C_s = \text{Rs.}0.75$ per unit short, $C_h = \text{Rs. } 8 * 15\% = \text{Rs. } 1.20$ and $C_o = \text{Rs.}15.00$ / order.

Using the formulas for Q^* and TC^* as

$$Q^* = \sqrt{\frac{2DC_o}{C_h} \left(\frac{C_s + C_h}{C_s} \right)} \quad TC^* = \sqrt{2DC_o C_h \left(\frac{C_s}{C_s + C_h} \right)}$$

We get $Q^* = 32$ units (approx.) and $TC^* = \text{Rs.}14.88$ (approx.)

Example 7.6 : A television manufacturing company produces its own speakers, which are used in the production of its television sets. The television sets are assembled on a continuous production line at rate of 8,000 per month. The company is interested in determining when and how much to procure, given the following information:

- (I) Each time a batch is produced, a set-up cost of Rs.12,000 is incurred.
- (II) The cost of keeping a speaker in stock is Rs. 0.30 per month.
- (III) The production cost of a single speaker is Rs.10.00 and can be assumed to be a unit cost.
- (IV) Shortage of a speaker, (if there exists) costs Rs. 1.10 per month.

Solution : Given $D = 8,000$ televisions per month, $C_o = \text{Rs.}12,000$ per production run, $C_h = \text{Rs.}0.30$ per unit per month, $C_s = \text{Rs.}1.10$ per unit short per month.

Case (i) When Shortages are not allowed, $Q^* = \sqrt{\frac{2DC_o}{C_h}} = 25,298$ speakers and $T = Q^*/D = 3.2$ months.

Thus, 25,298 speakers are to be produced every 3.2 months.

Case (ii) When shortages are permitted $Q^* = \sqrt{\frac{2DC_o}{C_h} \left(\frac{C_s + C_h}{C_s} \right)} = 28,540$ speakers and $T = Q^*/D = 3.6$ months

Hence, when shortages are permitted, 28,540 speakers are produced at every 3.6 months.

Optimal number of speakers stored = $Q^* \left(\frac{C_s}{C_s + C_h} \right) = 22,424$ speakers.

Thus, a shortage of 6,116 ($= 28,540 - 22,424$) speakers is permitted.

Or else optimal shortage level = $S^* = Q^* \left(\frac{C_h}{C_s + C_h} \right) = 6116$ (approx.)

Example 7.7 : A dealer supplies you the following information with regard to a product dealt in by him :
Annual demand = 5,000 units, ordering cost = Rs.25.00 per order, inventory carrying cost is 30% per unit per year of purchase cost Rs.100 per unit.

The dealer is considering the possibility of allowing some back-orders to occur for the product. He has estimated that the annual cost of back-ordering the product will be Rs.10.00 per unit.

- (I) What should be the optimum number of units of the product he should buy in one lot?
- (II) What quantity of the product should he allow to be back-ordered ?
- (III) How much additional cost will he have to incur on inventory if he does not permit back-ordering?

Solution : Given $R = 5,000$ units, $C_o = \text{Rs.}250$ per order, $C_h = 100 * 0.30 = \text{Rs.}30$ per unit and $C_s = \text{Rs.}10.00$ per unit.

$$Q^* = \sqrt{\frac{2DC_o}{C_h} \left(\frac{C_s + C_h}{C_s} \right)} = 288 \text{ units and } S^* = Q^* \left(\frac{C_h}{C_s + C_h} \right) = 72 \text{ units.}$$

$\therefore$ The units to be back-ordered $= Q - S = 216$ units.

$$\begin{aligned} \text{(ii)} \quad TC^* &= \sqrt{2DC_o C_h \left(\frac{C_s}{C_s + C_h} \right)} \\ &= \text{Rs.}2,165. \end{aligned}$$

If back-ordered are not permitted then total cost of an inventory system per unit is $\sqrt{2DC_o C_h} = \text{Rs.}8,660$.

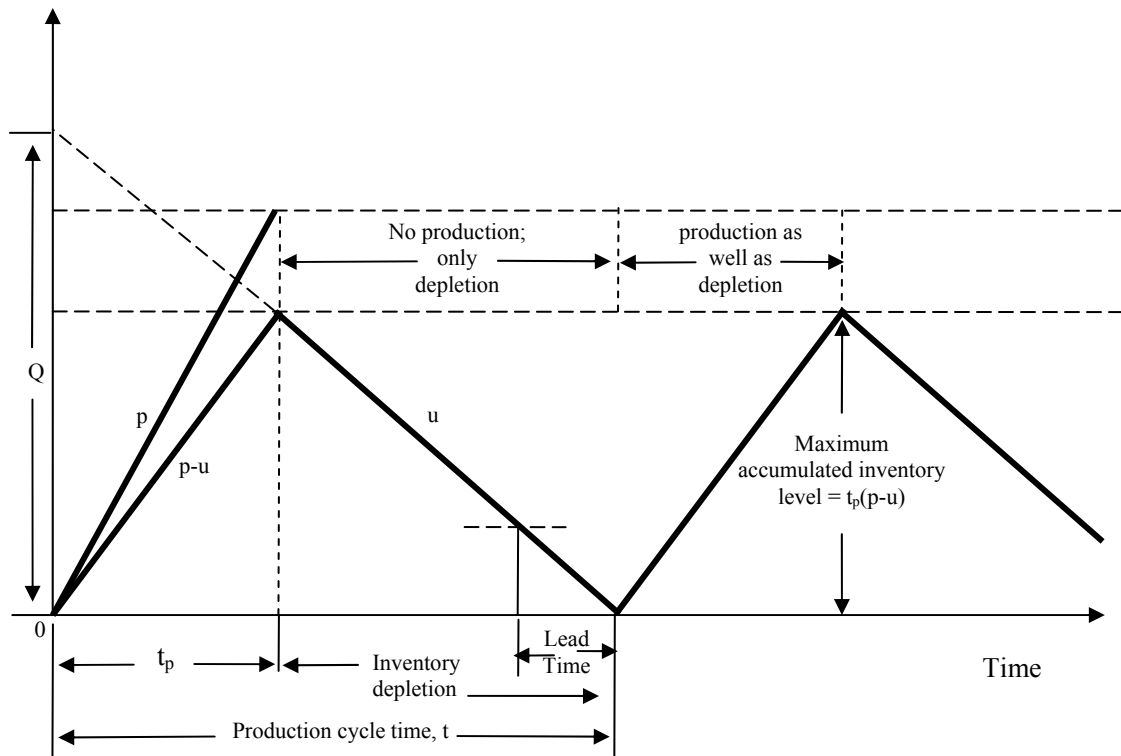
$\therefore$ Additional cost when back-orders are not permitted $= 8,660 - 2,164 = \text{Rs. } 6,495$.

7.7.3 Inventory model with finite replenishment rate (production rate), constant demand, and no shortages

Let us consider the situation in which a company supplies units to inventory at a uniform replenishment rate over time, rather than in economic order quantities at specific points of time. The amount ordered is not delivered all at once, but ordered quantity is sent or received gradually over a length of time at a finite rate per unit of time. This gradual replenishment situation occurs when a company is both a producer and user of an item or when it spreads its deliveries over time. Particularly in a manufacturing set up, a batch quantity that is internally ordered and after certain lead time the production process begins and a batch of items is produced over a period of time. The quantity produced is gradually consumed or used up till such time a reorder point is reached, when one more batch is ordered and the cycle starts once again.

In some cases usage and production (or delivery rates) are equal, and inventory will not build up because the company will use all items immediately. More typically, the production or delivery rate, p , will exceed the demand or usage rate, u ($p > u$).

The graphic depiction of this situation is shown in the following figure.



We start at zero inventory level. If t_p is the time period required to produce one entire batch amount Q at the rate p , then the rate at which the stocks arrive is $p = Q / t_p$.

The company uses and produces items during the first part of the inventory cycle i.e. during the time t_p the company produces a lot size of Q units. Since there is also a simultaneous usage, the inventory level builds gradually at the rate $p-u$ units during this time. This production stops when a batch size of Q units is produced. When production ceases, the second part of the inventory cycle begins. The company will deplete its inventory level at the usage, or demand, rate u .

In cases when units are ordered from outside sources, at a desired reorder level, according to a constant known lead time, order is placed so that as soon as the on hand inventory level reaches zero, the units are received in the system. In case when the goods are manufactured internally, this reorder level is fixed so as to set up the machines for a new production run so that as soon as the on hand inventory level reaches zero, the production starts.

The maximum inventory level reached at the end of t_p = inventory accumulation rate $\times$ production time

$$= (p-u)t_p = (p-u)\frac{Q}{p} = \left(1-\frac{u}{p}\right)Q$$

The inventory holding cost IHC = Average inventory $\times$ cost of holding one unit = $\left(1-\frac{u}{p}\right)\frac{Q}{2}C_h$

The annual set up or the ordering cost = $(D/Q) C_o$

Total Inventory Cost TC = Ordering cost + holding cost = $\frac{D}{Q}C_o + \left(1-\frac{u}{p}\right)\frac{Q}{2}C_h$

Differentiating this equation of TC with respect to Q and solving for Q, we get the optimal lot size Q* as

$$Q^* = \sqrt{\frac{2DC_o}{C_h} \left(\frac{p}{p-u} \right)}$$

The length of time required to produce a lot is $t_p^* = Q^* / p$

$$\text{The maximum inventory level} = Q^* \left(1 - \frac{u}{p} \right)$$

$$\text{The length of time required to deplete the maximum on-hand inventory } t_d^* = \frac{Q^*}{u} \left(1 - \frac{u}{p} \right)$$

$$\text{The length of an inventory cycle } t = t_p^* + t_d^* = \frac{Q^*}{p} + \frac{Q^*}{u} \left(1 - \frac{u}{p} \right) = \frac{Q^*}{u}$$

$$\text{The minimum total inventory cost } TC^* = \frac{D}{Q^*} C_o + \left(1 - \frac{u}{p} \right) \frac{Q^*}{2} C_h$$

$$\text{When the value of } Q^* \text{ is substituted, we get } TC^* = \sqrt{2DC_o C_h \left(1 - \frac{u}{p} \right)}$$

Example 7.7 : A product is to be manufactured on a machine. The cost, production, demand, etc are as follows :

Ordering cost per order = Rs.30,

Purchase cost per unit = Rs.0.10

Inventory holding cost per unit per annum = Rs.0.05

Production rate = 1,00,000 units per year

Demand rate = 10,000 units / year.

Determine the economic manufacturing quantity.

Solution : Given : C_o = Rs. 30 per order, C = Rs.0.10 per unit, C_h = 0.05 / unit / annum, p = 1,00,000 units per year and u = 10,000 units per year

$$Q^* = \sqrt{\frac{2DC_o}{C_h} \left(\frac{p}{p-u} \right)}$$

$$= 3,561 \text{ units}$$

Example 7.8 : A contractor has to supply 10,000 bearings per day to an automobile manufacturer. He finds that, when he starts a production run, he can produce 25,000 bearings per day. The cost of holding a bearing in stock for one year is Rs.2 and the set-up cost of a production run is Rs.1,800. How frequently should production run be made?

Solution : Given : $C_h = \text{Rs.}2.00 / \text{bearing} / \text{year} = \text{Rs.}0.0055 / \text{bearing} / \text{year}$, $C_o = \text{Rs.}1800 / \text{production run}$,

$u = 10,000 \text{ bearings/day}$, $p = 25,000 \text{ bearings} / \text{day}$. Then using the above formula for Q^* , we get

$Q^* = 1,04,447 \text{ bearings}$.

$T = Q^* / u = 10.5 \text{ days}$.

Length of production cycle = $t_1 = Q^* / p = 4 \text{ days}$.

Thus, the production cycle starts at an interval of 10.5 days and production continues for 4 days.

Example 7.9 : Find the most economic batch quantity of a product on a machine if the production rate of that item on the machine is 200 pieces per day and the demand is uniform at the rate of 100 pieces per day. The ordering cost is Rs. 200 per batch and the cost of holding one item in inventory is Rs.0.81 per day. How will the batch quantity vary if the production rate is infinite?

Solution : Given : $C_o = \text{Rs.}200 / \text{order}$, $C_h = \text{Rs.}0.81 / \text{unit} / \text{day}$, $u = 100 \text{ units} / \text{day}$, $p = 200 \text{ units} / \text{day}$. Using the above formula for Q^* we get, $Q^* = 317 \text{ units}$.

Cycle time, $T = Q^* / u = 3.17 \text{ days}$.

Length of production cycle, $t_1 = Q^* / p = 1.58 \text{ days}$.

If the production rate is infinite, i.e. $P \rightarrow \infty$ then $Q^* = \sqrt{\frac{2DC_o}{C_h}} = 222 \text{ units}$.

7.8 THE CONTINUOUS REVIEW MODEL: When to order

Here we consider a **continuous review inventory** system. Here the inventory level is being monitored on a continuous basis so that a new order can be placed as soon as the inventory level drops to the reorder point.

Personals, in practice, make a physical count of inventory at periodic intervals to decide how much of each item to order. Using the continuous review system to determine when to reorder, we review the remaining quantity of an item each time a withdrawal is made from inventory. In practice, operations managers make a physical count of inventory at periodic intervals (daily, weekly, or monthly) to decide how much of each item to order. Many small retailers use this approach, simply checking the quantities on shelves and in the storeroom on a periodic basis. Another very elementary type of continuous review system is the traditional **two-bin system**, which sets aside two containers, or bins, to hold the total inventory of an item. Items are withdrawn from the first bin until it is empty, at which point it is time to reorder the quantity that will again fill the bin. The second bin contains enough stock to satisfy demand until the order comes in, plus an extra amount to provide a cushion against a stock out.

In recent years, two-bin systems have been largely replaced by computerized inventory systems. Each addition to inventory and each sale causing a withdrawal are recorded electronically, so that the current inventory level always is in the computer. Therefore, a computer will trigger a new order as soon as the inventory level has

dropped to the reorder point. The continuous review system is also called a **reorder point (ROP) system**, or a **fixed order quantity system**. It is also called a **(R,Q) policy**. It works this way:

“Place an order for Q units whenever a withdrawal brings the inventory to the reorder point R .”

The continuous review system has only two parameters, Q and R , and each new order is of size Q . For a manufacturer managing its finished products inventory, the order will be for a production run of size Q . For a wholesaler or retailer, the order will be a purchase order for Q units of the product.

Let us address the question of “*when to order*”. At the time of placing a new order, the stock in hand should be sufficient to meet the demand until the new order arrives.

When both demand and the lead time are deterministic (known with certainty), the reorder level is calculated as:

$$\text{Reorder level (ROL)} = \text{Demand during the replenishment lead time} = d \times LT$$

Example : Demand for an item is 5200 units a year and the EOQ has been calculated as 250 units. If the lead time is 2 weeks, then the ordering policy will be $\text{ROL} = (5200 / 52) \times 2 = 200$ units.

This means that as soon as the stock level falls to 200 units an order equal to $\text{EOQ} = 250$ units should be placed. This rule of ordering is applicable only if the lead time is shorter than the stock cycle. Here, the stock cycle is $T = Q^*/d = 250/100 = 2.5$ weeks.

But if the lead time is 3 weeks, then the $\text{ROL} = 100 \times 3 = 300$ units. Since $\text{EOQ} = 250$ units, therefore stock level varies between zero and 250 units. Thus lead time demand of 300 units suggests that there should be one outstanding order. In such cases, an order is placed when

$$\text{Lead time demand} = \text{stock on hand} + \text{stock on order}$$

$$300 = \text{stock on hand} + 250$$

$$\text{or} \quad \text{ROL} = \text{Lead time demand} - \text{stock on hand}$$

In general, an ordering policy is stated as: “when lead time is between $n \times T$ and $(n+1) \times T$, order an amount Q^* whenever stock on hand falls to $d \times LT - n \times Q^*$, where n is number of stock cycle and lead time exceeds cycle time T ”. For example, lead time of 3 weeks is between 2 and 3 stock cycles, so $n = 2$, then

$$\text{ROL} = d \times LT - n \times Q^*$$

$$= 100 \times 3 - 2 \times 250$$

$$= 50 \text{ units.}$$

i.e each time the stock on hand declines to 50 units, an order of 250 units is placed.

Example 7.10 : The annual demand for a product is 3,600 units with an average of 12 – units per day. The lead-time is 10 days. The ordering cost per order is Rs.10 and the annual carrying cost is 25% of the value of the inventory. The price of the product per unit is Rs. 3.00.

- (I) What will be the EOQ?
- (II) Find the purchase cycle time?
- (III) Find the total inventory cost per year?
- (IV) If the safety stock of 100-units is considered necessary, what will be the reorder level and the total annual cost of inventory which will be relevant to inventory decision?

Solution : Given $D = 3,600$ units, $C_o = \text{Rs. } 20$ per order, $C_h = \text{Rs. } 3 * 25\%$ per unit per year and lead-time = 10 days. Since, the demand is uniform at 12 – units per day, the total number of working days in the year = $3600/12 = 300$.

The basic EOQ formula gives $Q = 438$ (appro.)

(ii) Cycle time = $438 / 12 = 36.5$ days.

(iii) $TC = CD + (D C_o / Q) + (C_h Q / 2) = \text{Rs. } 11128.60$.

(iv) Re-order level = Safety stock + lead-time demand = $100 + 12 * 10 = 220$ units.

$\therefore$ Average inventory = Safety stock + $Q / 2 = 319$ units.

$\therefore TC = (D C_o / Q) + (C_h Q / 2) = \text{Rs. } 164.38$.

7.9 Operation Of Periodic Review System:

This system is also known as the fixed interval system or replenishment inventory system or cyclic review system.

In this system, the size of order quantity may vary with the fluctuation in demand, but the ordering interval is fixed. The system is specified for any item by

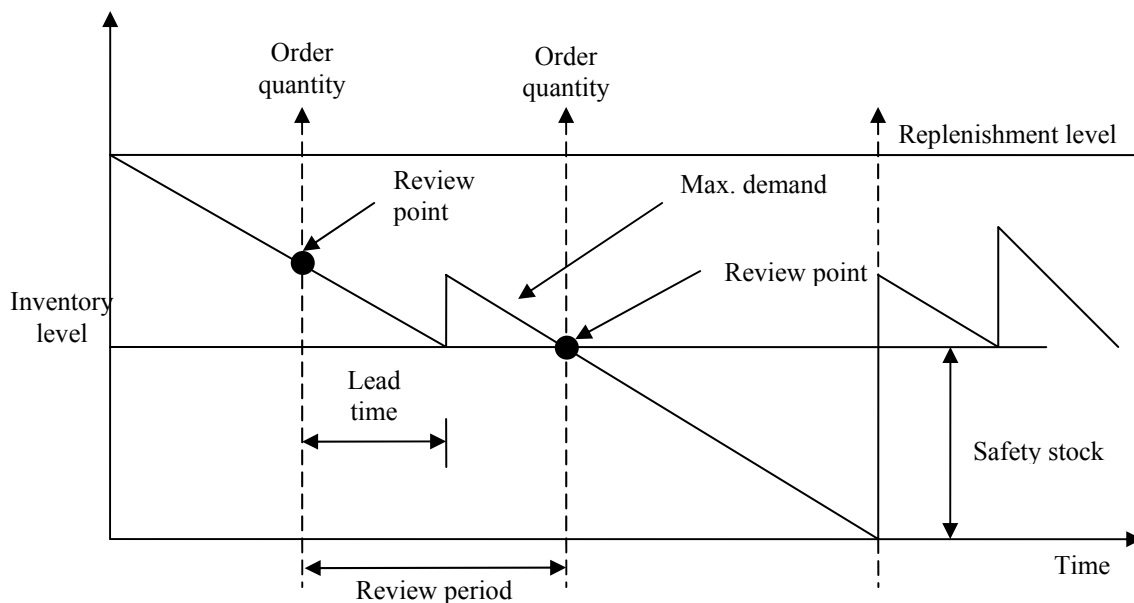
(1) review period t , and (2) replenishment level, or reorder level, R

In this system, the inventory position is periodically reviewed – once, weekly, monthly, quarterly or half-yearly. At each review period, an order is placed for an amount equal to the difference between a fixed replenishment level and the actual inventory level. The calculation of R is based on the formula:

Replenishment level $R = \text{Average consumption during a review period} + \text{Lead time} + \text{safety stock}$

Order quantity = Replenishment level – stock available

The following diagram gives the way in which the periodic review system operates.



The order quantity is variable in size from one review date to another.

Example 7.11: For a periodic review system, find out the various parameters for an item with the following data:

Annual consumption = 14000 units, Cost of one unit = Rs. 10, Supplier's minimum quantity = 1000 units, normal lead time = 10 days, maximum lead time = 15 days, maximum consumption = 1020 (average consumption).

Solution: The maximum number of orders to cover the annual requirement is $14000 / 1000 = 14$ orders.

Therefore, the review period should be $1/14^{\text{th}}$ of an year or 26 days. We consider the period of review as 1 month instead of 26 days.

Now,

$$\begin{aligned} \text{Safety stock} &= \text{Max. consumption rate (review period + max. lead time)} - \text{normal consumption rate (review period + normal lead time)} \\ &= 1.20 \times (14000/12) (1 + (15/30)) - (14000/12) (1 + (10/30)) \\ &= 545 = \text{approx. 550 units} \end{aligned}$$

$$\begin{aligned} \text{Reorder level or replenishment level } R &= \text{Average consumption rate} \times (\text{review period} + \text{normal lead time}) + \text{safety stock} \\ &= (14000/12) (1 + (10/30)) + 550 = 2105 \text{ units.} \end{aligned}$$

$$\begin{aligned} \text{Maximum inventory when the supplies are received} &= 500 + \text{order quantity} \\ &= 550 + (14000/12) = 1710 \text{ units} \end{aligned}$$

$$\text{Maximum inventory} = 550$$

$$\text{Average inventory} = (1710 + 550)/2 = 1130 \text{ units.}$$

Probabilistic EOQ Model:

Stochastic or probabilistic inventory models are designed for analyzing inventory systems where there is a considerable uncertainty about future demands. In such cases the demand is described by a probability distribution. The models can further be categorized broadly under *continuous* and *periodic* review situations. In this chapter we will consider only stochastic single period models without initial stock and without set up cost.

7.10 Stochastic Single – Period Models:

There are certain products which can be carried in inventory for only a very limited period of time before it can no longer be sold. Such products are perishable products. For such products models called single period models are designed. Following are such perishable products for which single period models are suitable:

Flowers, fresh foods, fruits, vegetables, fashion goods, seasonal goods, etc.

One important example of a perishable product is a daily news paper being sold at a news stand. A particular day's news paper can be carried in inventory for only a single day before it becomes outdated and needs to be replaced by the next day's news paper. When the demand for the news paper is a random variable, the owner of the news stand needs to choose a daily order quantity that provides an appropriate trade-off between the potential cost of over ordering (the wasted expense of ordering more than that can be sold) and the potential cost of under ordering (the lost profit from ordering fewer than that can be sold). This problem is the classical **newsboy problem**. Mathematically, it is termed as the **single period stochastic or probabilistic model**.

Model without set up cost:

Here we assume the following additional notations:

K = Set up cost per order

D = Probabilistic demand during the period

$p(D)$ = probability density function of the demand during the period

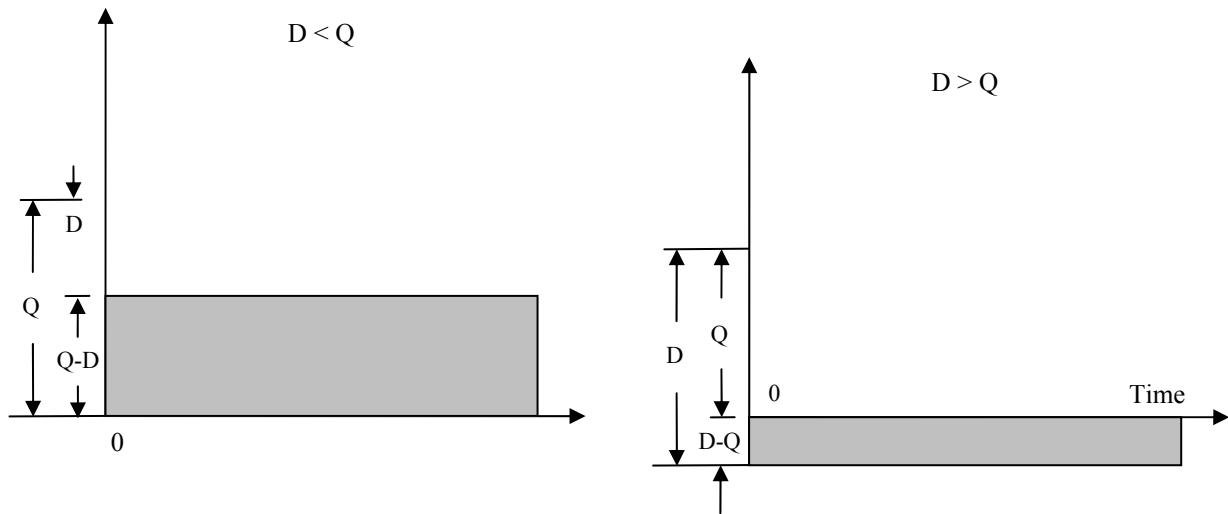
Q = order quantity

x = the amount on hand before an order is placed

The assumptions are:

1. The model refers to a single perishable product involving a single time period because it cannot be sold later
2. Demand is continuous and a random variable. The probability distribution of D is known
3. Demand occurs instantaneously at the start of the period immediately after the order is received.
4. No set up cost is incurred
5. There is no initial inventory on hand

The following figure depicts the inventory situation after the demand D is satisfied. If $D < Q$, the quantity $Q - D$ is held during the period. Otherwise, if $D > Q$, a shortage amount $D - Q$ will result.



The Expected cost for the period, $ETC(Q)$, is expressed as

$ETC(Q)$ = expected purchase (production) costs + expected inv. Holding cost + expected shortage cost

$$ETC(Q) = C(Q - x) + C_h \int_0^Q (Q - D)p(D)dD + C_s \int_Q^{\infty} (D - Q)p(D)dD$$

Taking the first derivative of $ETC(Q)$ with respect to Q and equating it to zero, we get

$$\begin{aligned} C + C_h \int_0^Q p(D)dD - C_s \int_Q^{\infty} p(D)dD &= 0 \\ C + C_h P(D \leq Q) + C_s (1 - P(D \leq Q)) &= 0 \\ P(D \leq Q^*) &= \frac{C_s - C}{C_s + C_h} \end{aligned}$$

The right hand side of the above formula is called the *critical ratio*. The value of Q^* is defined only if the critical ratio is non-negative i.e $C_s \geq C$. Here we say that

$C_s - C$ = unit cost of under ordering

= decrease in the profit that results from failing to order a unit that could have been sold during the period.

$C + C_h$ = unit cost of over ordering

= decrease in profit that results from ordering a unit that could not be sold during the period.

In the above case the first assumption was that demand is continuous. If the demand is discrete, then $p(D)$ is defined at only discrete points and the cost function is defined as

$$ETC(Q) = C(Q - x) + C_h \sum_{D=0}^Q (Q - D)p(D) + C_s \sum_{D=Q+1}^{\infty} (D - Q)p(D)$$

The necessary optimality conditions are

$$ETC(Q-1) \geq ETC(Q) \quad \text{and} \quad ETC(Q+1) \geq ETC(Q)$$

which further boils down to

$$P(D \leq Q^* - 1) \leq \frac{C_s - C}{C_s + C_h} \leq P(D \leq Q^*)$$

Example 7.12 : The probability distribution of monthly sale of a certain item is as follows :

Monthly Sales :	0	1	2	3	4	5	6
Probability :	0.01	0.06	0.25	0.35	0.20	0.03	0.10

The cost of carrying inventory is Rs. 30 per unit per month and the cost of unit short is Rs. 70 per month. Determine the optimum stock level which minimizes the total expected cost.

Solution :

Given C_h = Rs. 30 per unit per month and C_s = Rs. 70 per month.

Then the critical ratio as per the above equation $C_s / (C_h + C_s) = 0.7 / (0.7 + 0.3) = 0.7$

We now compute the cumulative probability from the data given

Monthly Sales	Probability	Cumulative Probability
0	0.01	0.01
1	0.06	0.07
2	0.25	0.32
3	0.35	0.67
		← 0.7
4	0.20	0.87
5	0.03	0.90
6	0.10	1.00

Clearly, $0.67 < 0.7 < 0.87$. So using equation (11.75), $S = 4$ is the optimum order level.

REVIEW EXERCISE

Q. An aircraft company uses rivets at an approximate demand rate of 2,500 kg per year. Each unit costs Rs. 30 per kg and the company personnel estimate that it costs Rs. 130 to place an order, and that the carrying cost of inventory is 10 % per year. How frequently should orders for rivets be placed? Also determine the optimum size of each order.

Ans : $Q = 466$ units (approx.), $T = 0.18$ year, $n = 5$ orders / year.

Q. The annual demand for an item is 3,200 units. The unit cost is Rs. 6 per unit and inventory carrying charges is 25 % per annum. If the cost of one procurement is Rs. 150, determine (i) EOQ, (ii) number of orders per year, (iii) time between two consecutive orders, and (iv) the optimal cost.

Ans : $Q = 800$ units, $n = 4$, $T = 3$ months, $TC = \text{Rs. } 20,400$

Q. The annual requirements for a particular raw material are 2,000 units, costing Rs. 1.00 each to the manufacturer. The ordering cost is Rs. 10.00 per order and the carrying cost is 16 % per annum of the average inventory value. Find the economic order quantity and the total cost of an inventory system.

Ans : $Q = 500$ units, $TC = \text{Rs. } 80.00$

Q. For an item, the production is instantaneous. The holding cost of one item is Rs. 1.00 per month and the set-up cost is Rs. 25 per run. If the demand is 200 units per month, find the optimum quantity to be produced per set-up and total cost of storage and set-up per month.

Ans : $Q = 100$ units, Total cost of storage and set-up = Rs. 125

Q. A contract has a requirement for cement that amounts to 300 bags per day. No shortages are allowed. Cement costs Rs. 2.00 per bag, inventory carrying cost is 10 % of the average inventory valuation per day and it costs Rs. 20 to purchase order. Find the minimum cost of purchase quantity?

Ans : $Q = 100$ units, $TC = \text{Rs. } 120.00$

Q. A manufacturer has to supply his customer with 600 units of his product per year. Shortages are not allowed and the inventory holding cost is 60 paise per unit per year. The set-up cost per run is Rs. 80.00. Find (i) the economic order quantity, (ii) the minimum average yearly cost, (iii) the optimum number of order per year, (iv) the optimum period of supply per optimum order, and (v) the increase in the total cost associated with ordering (a) 20 % more and (b) 40 % less than EOQ.

Ans : $Q = 400$ units, $TC = \text{Rs. } 240$, $n = 3/2$, $T = 2/3$ yr., Rs. 4.00.

Q. Kissan manufactures 50,000 bottles of tomato ketchup in a year. The factory cost per bottle is Rs. 5.00, the set-up cost per production run is estimated to be Rs. 90 and the carrying costs on finished goods inventory is 20

% of the unit cost per annum. The production rate is 600 bottles per day and sales 150 bottles per day. What is the optimal production lot-size and number of production runs?

Q. The annual demand for a product is 1,00,000 units. The rate of production is 2,00,000 units per year. The set-up cost per production run is Rs. 5,000 and the variable production cost of each item is Rs. 10. The annual holding cost per unit is 20 % of its value. Find the optimum production lot-size and the length of the production run?

Ans : $Q = 31,600$ units, $T = 0.316$ yrs.

Q. A contractor undertakes to supply diesel engines to a truck manufacturer at a rate of 25 per day. He finds that the cost of holding a completed engine in stock is Rs. 16 per month, and there is a clause in the contract penalizing him Rs. 10 per engine per day late for missing the scheduled delivery date. Production of engines is in batches, and each time a new batch is started there are set-up costs of Rs. 10,000. How frequently should batches be started, and what should be the initial inventory level at the time each batch is completed?

Ans : $Q = 994$ engines (approx.), $T = 40$ days (approx.)

Q. A commodity is to be supplied at a constant rate of 200 units per day. Supplies of any amounts can be had at any required time, but each ordering costs Rs. 50.00, cost of holding the commodity in inventory is Rs. 2.00 per unit per day while the delay in the supply of the item induces a penalty of Rs. 10.00 per unit per delay of one day. Find optimum order-level and reorder cycle time. What would be the best policy if the penalty cost becomes ∞ ?

Ans : $S = 109$ units (approx.), $T = \frac{1}{2}$ day, $Q = 100$ units

Q. The cost of parameters and other factors for a production inventory system of automobile pistons are given below. Find (i) Optimum lot-size, (ii) number of shortages and (iii) manufacturing time and time between two set-ups.

Demand per year = 6000 units; unit cost = Rs. 40;

Set-up cost = Rs. 500; production rate per year = 36,000 units

Holding cost per year = Rs.8; Shortage cost per unit per year = Rs. 20

Ans : $Q = 1,123$ units, $S = 267$ units, $t_1 = 0.03$ yrs $T = 0.19$ yrs.

Q. In a central grain store, it takes about 15 days to get the stock after placing an order and daily 500 tons are dispatched to neighbouring markets. On an ad-hoc basis safety stock is assumed to be 10 day's stock. Calculate reorder point.

Ans : Reorder point = 12,500 tons.

Q. (a) Minicomputer company purchases a component of which it has a steady usage of 1,000 units per year. The ordering cost is Rs. 50 per order. The estimated cost of money invested in inventory is 25 % per year. The unit cost of component is Rs. 40. Calculate the optimal ordering policy and total cost of the inventory system, including purchase cost of the components.

Ans : $Q = 100$ units, $TC = \text{Rs. } 41,000$

Q. A TV dealer find that cost of holding a television set in stock for a week is Rs. 20; customers who can not obtain a new television set immediately tend to go to other dealers, and he estimates that for every customer who does not get immediate delivery he loses on an average Rs. 200. For one particular model of television the probabilities for a demand of 0, 1, 2, 3, 4, and 5 television sets in a week are 0.05, 0.10, 0.20, 0.30, 0.20 and 0.15, respectively. How many television sets per week should the dealer order? (Assume that there is no lead-time).

Ans : $Q = 4$ TV sets / week.