INDUSTRIAL STATISTICS AND OPERATIONAL MANAGEMENT

5 : PROJECT MANAGEMENT

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CHAPTER 5

PROJECT MANAGEMENT

5.1 Introduction:

At one point or another almost every organization will take on a large and complex project. A construction company putting up an office building or laying a highway must complete thousands of costly activities. A shipyard requires tens of thousands of steps in constructing an oceangoing tugboat. An oil refinery about to shut down for a major maintenance project faces astronomical expenses if this difficult task is unduly delayed for any reason. Almost every industry worries about how to manage similar large-scale, complicated projects effectively.

Large, often one-time, projects are difficult challenges to operations managers. The stakes are high. Millions in cost overruns have been wasted due to poor planning on projects. Unnecessary delays have occurred due to poor scheduling. And companies have gone bankrupt due to poor controls.

Special projects that take months or years to complete are usually developed outside the normal production system. Project organizations within the firm are set up to handle such jobs and are often disbanded when the project is complete. The management of large projects involves there phases.

1. planning ; 2. scheduling ; 3. control.

Project Planning 1. Setting goals 2. Defining the project 3. Tying needs into timed project activities 4. Organizing the term	**Tools** Time and cost estimates Budgets Cash flow charts Material availability details Personnel data charts Engineering Diagrams
Project Scheduling 1. Typing resources (people, money supplies) to specific activities 2. Relating activities to each other 3. Updating and revising on regular basis	**Tools** CPM and PERT Milestone charts Cash flow schedules Gantt charts
Project Controlling 1. Monitoring resources, costs, quality budgets. 2. Revising and changing plans 3. Shifting resources to meet time, cost, and quality demands.	**Tools** PERT charts Reports describing 1. budgets by department 2. delayed activities 3. slack activities 4. quality of work completed

Table 5.1

5.2 Project Planning :

Projects can usually be defined as series of related tasks directed towards a major output. A new organization form, developed to make sure existing programs continue to run smoothly on a day-to-day basis while new projects are successfully completed, is called ***project organization***.

A project organization is an effective way of pooling the people and physical resources needed for a limited time to complete a specific project or goal. It is basically a temporary organization structure designed to achieve results by using specialists from throughout the firm.

The project organization works best when;

1. Work can be defined with a specific goal and deadline;
2. The job is unique or somewhat unfamiliar to the existing organization;
3. The work contains complex interrelated tasks requiring specialized skills;
4. The project is temporary but critical to the organization.

5.3 Project Scheduling :

Project scheduling is determining the project's activities in the time sequence in which they have to be performed. Materials and people needed at each stage of production are computed in this phase, and the time each activity will take is also set. Separate schedules for personnel need by type of skill (management, engineering) are charted. Charts can also be developed for scheduling materials. One popular project scheduling approach is the Gantt chart (named after Henry Gantt).

Whatever the approach taken by a project manager, project scheduling serves several purposes:

1. It shows the relationship of each activity to others and to the whole project.
2. It identifies the precedence relationships among activities.
3. It encourages the setting of realistic time and cost estimates for each activity.
4. It helps make better use of people, money, and material resources by identifying critical bottlenecks in the project.

5.4 Project Controlling :

The control of large projects, like the control of any management system, involves close monitoring of resources, costs, quality and budgets. Control also means using a feedback loop to revise the project plan and having the ability to shift resources to where they are needed most.

Management of big projects that consist of a large number of activities pose complex problems in *planning, scheduling and control*, especially when the project activities have to be performed in a specified technological sequence. With the help of PERT (Program Evaluation and Review Technique), and CPM (Critical Path Method), the project manager can

1. Plan the project ahead of time and foresee possible sources of troubles and delays in completion.
2. Schedule the project activities at the appropriate times to conform with proper job sequence so that the project is completed as soon as possible.
3. Coordinate and control the projects activities so as to stay on schedule in completing the project.

Thus both PERT and CPM are aids to efficient project management. They differ in their approach to the problem and the solution technique. The nature of the project generally dictates the proper technique to be used.

5.5 Origin and Use of PERT :

PERT was developed in the U.S. Navy during the late 1950s to accelerate the development of the Polaris Fleet Ballistic Missile. The development of this weapon involved the coordination of the work of thousands of private contractors and other government agencies. The coordination by PERT was so successful that the entire project was completed 2 years ahead of schedule. Nowadays it is extensively used in industries and other service organizations as well.

The time required to complete the various activities in a research and development project is generally not known *a priori*. Thus PERT incorporates uncertainties in activity times in its analysis. It determines the probabilities of completing various stages of the project by specified deadlines. It also calculates the expected time to complete the project. An important and extremely useful by product of PERT analysis is its identification of various “bottlenecks” in a project. In other words, it identifies the activities that have high potential for causing delays in completing the project on schedule. Thus, even before the project has started, the project manager knows where he or she can expect delays. The manager can then take the necessary preventive measures to reduce possible delays so that the project schedule is maintained.

Because of its ability to handle uncertainties in job times, PERT is mostly used in research and development projects.

5.6 Origin and Use of CPM :

Critical Path Method closely resembles PERT in many aspects but was developed independently by E.I. du Pont de Nemours Company. Actually, both techniques, PERT and CPM, were developed almost simultaneously. It was developed to have a better planning in controlling the overhaul and maintenance of chemical plants. The major difference between the two techniques is that CPM does not incorporate uncertainties in job times. Instead it assumes that activity times are proportional to the amount of resources allocated to them, and by changing the level of resources the activity times and the project completion time can be varied. Thus CPM assumes prior experience with similar projects from which the relationships between resources and job times are available. CPM then evaluates the trade-off between project costs and project completion time.

CPM is mostly used in construction projects where there is prior experience in handling similar projects.

5.7 Applications of PERT and CPM :

A partial list of applications of PERT and CPM techniques in project management is as follows:

1. Construction projects (e.g., buildings, highways, houses, and bridges)
2. Preparation of bids and proposals for large projects.
3. Maintenance planning of oil refineries, ship repairs, and other large operations.
4. Development of new weapons systems and new manufactured products.
5. Manufacture and assembly of large items such as airplanes, ships, and computers.
6. Simple projects.

5.8 Framework of PERT and CPM :

Six steps are common to both PERT and CPM. The procedure is as follows:

1. Define the projects and all of its significant activities or tasks.
2. Develop the relationships among the activities. Decide which activities must precede and which must follow others.
3. Draw the network connecting all of the activities.
4. Assign time and/or cost estimates to each activity.
5. Compute the longest time path through the network; this is called the *critical path***.**
6. Use the network to help plan, schedule, monitor and control the project.

Step 5, finding the critical path, is a major part of controlling a project. The activities on the critical path represent tasks that will delay the entire project if they are delayed. Managers derive flexibility by identifying noncritical activities and replanning, rescheduling, and reallocating resources such as labor and finances.

Although PERT and CPM differ to some extent in terminology and in the construction of the network, their objectives are the same. Furthermore, the analysis used in both techniques is very similar. The major difference is that PERT employs three time estimates for each activity. Each estimate has an associated probability of occurrence, which, in turn, is used in computing expected values and standard deviations for the activity times. CPM makes the assumption that activity times are known with certainty, and hence only one time factor is given for each activity.

PERT and CPM are important because they can help answer questions such as the following about projects with thousands of activities.

1. When will be entire project be completed?
2. What are the critical activities or tasks in the project, that is, the ones that will delay the entire project if they are late?
3. Which are the noncritical activities, that is, the ones that can run late without delaying the whole project's completion?
4. What is the probability that the project will be completed by a specific date?
5. At any particular date, is the project on schedule, behind schedule, or ahead of schedule?
6. On any given date, is the money spent equal to, less than, or greater than the budgeted amount?
7. Are there enough resources available to finish the project on time?
8. If the project is to be finished in a shorter amount of time, what is the best way to accomplish this at the least costs?

5.9 Constructing the Project Network :

Analysis by PERT/CPM techniques uses the network formulation to represent the project activities and their ordering relations. Construction of a project network is done as follows:

1. Arcs in the network represent individual jobs or activities in the project.
2. Nodes represent specific points in time which mark the completion of one or more jobs in the project.
3. Direction on the arc is used to represent job sequence. It is assumed that any job directed towards a node must be completed before any job directed away from that node can begin.

We illustrate the construction of project networks with a few examples.

Example 5.1 Given the following information, develop a network

ACTIVITY	IMMEDIATE PREDECESSOR (S)
A	-
B	-
C	A
D	B

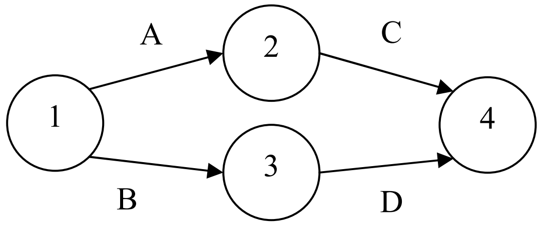


Fig 5.1

You will note that we assigned each event a number. As you will see later, it is possible to identify each activity with a beginning and ending event or node. For example, activity A in fig. 5.1 is the activity that starts with event 1 and ends at node, or event, 2. In general, we number node from left to right. The beginning node, or event, of the entire project is number1, while the last node, or event, in the entire project bears the largest number. The last node shows the number 4.

Example 5.2 Consider seven jobs *A, B, C, D, E, F* and *G* with the following job sequence:

Job A precedes B and C
Jobs C and D precede E
Job B precedes D
Job E and F precede G

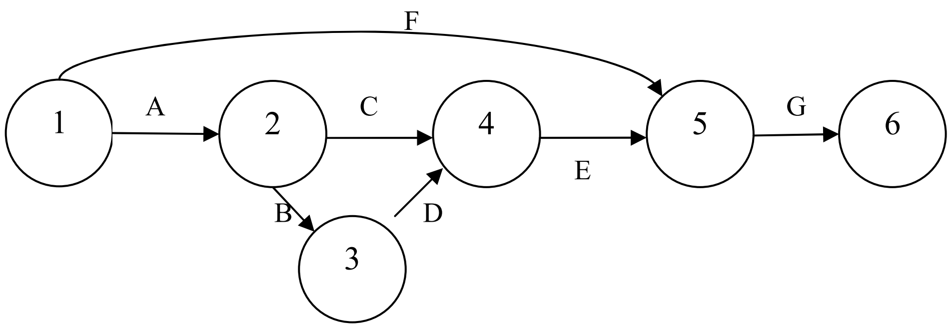


Fig 5.2

In the network (fig. 5.2), every are *(i,j)* represents a specific job in the project. Node 1 represents the start of the project, whereas node 6 denotes the project's completion time. The intermediate nodes represent the completion of various stages of the project. The nodes of the project network are generally called events.

Event : An ***event*** is a specific point in time that marks the completion of one or more activities, well recognizable in the project.

We can also specify network by events and the activities that occur between events. The following example shows how to develop a network based on this type of specification scheme.

Example 5.3 Given the following table, develop a network

BEGINNING EVENT	ENDING EVENT	ACTIVITY
1	2	1-2
1	3	1-3
2	4	2-4
3	4	3-4
3	5	3-5
4	6	4-6
5	6	5-6

Beginning with the activity that starts at event 1 and ends at event 2, we can construct the following network.

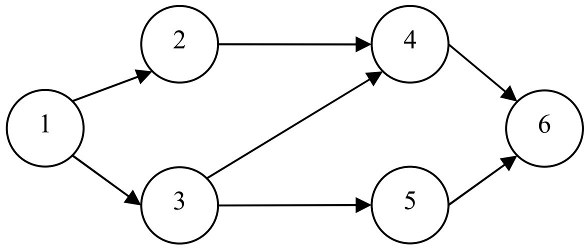


Fig 5.3

All that is required to construct a network is the starting and ending event for each activity.

5.10 Dummy Activities and Events :

You may encounter a network that has two activities with identical starting and ending events. Dummy activities and events can be inserted into the network to deal with this problem. The use of dummy activities and events is especially important when computer programs are to be employed in determining the critical path, project completion time, project variance, and so on. Dummy activities and events can also ensure that the network properly reflects the project under consideration. The following example illustrates the procedure.

Example 5.4 Develop a network based on the following information :

ACTIVITY	IMMEDIATE PREDECESSOR (S)	ACTIVITY	IMMEDIATE PREDECESSOR (S)
A	-	E	C, D
B	-	F	D
C	A	G	E
D	B	H	F

Given these data, you might develop the following network (fig. 5.4).

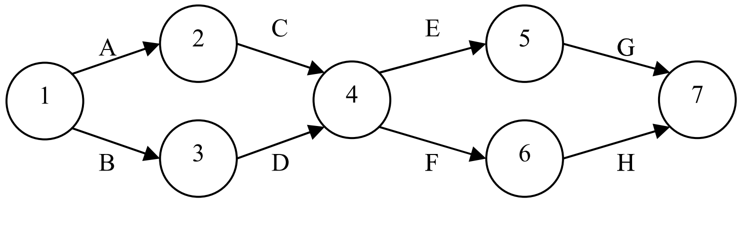


Fig 5.4

Look at activity F. According to the network, both activities C and D must be completed before we can start F, but in reality, only activity D must be completed (see the table), Thus the network is not correct. The addition of a dummy activity and a dummy event can overcome this problem, as shown fig. 5.5.

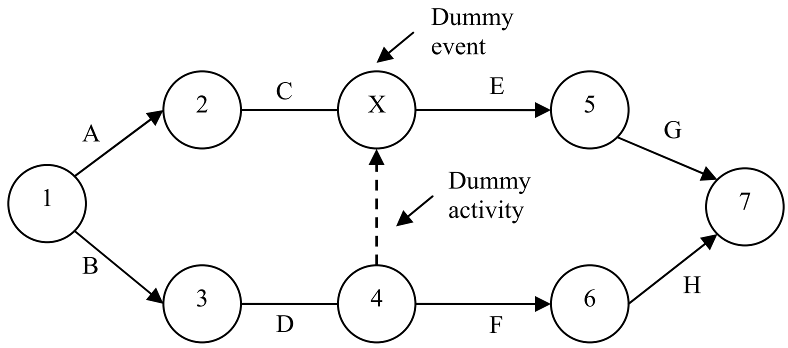


Fig 5.5

Now the network embodies all of the proper relationships and can be analyzed as usual.

A dummy activity has a completion time, *t* of zero.

Example 5.5 Consider a project with five jobs *A, B, C, D* and *E* with the following job sequence:

Jobs A precedes C and D,

Job B precedes D,

Job C and D precedes E.

The completion times for A, B, C, D and E are 3, 1, 4, 2, 5 days, respectively.

The project network is shown in the figure 15.6.

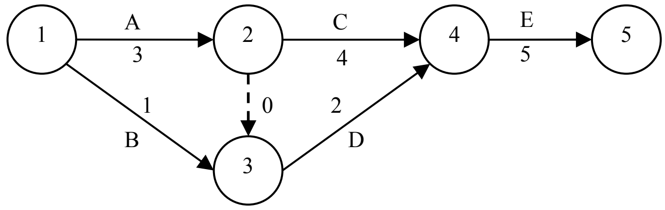


Fig 5.6

Arc (2, 3) (dotted line in the figure) represents a dummy job that does not exist in reality in the project. The dummy activity is necessary so as to avoid ambiguity in the job sequence. The completion time of the dummy job is always zero, and it is added in the project network whenever we want to avoid an arc *(i,j)* representing more than one job in the project. In the figure, event 3 represents the completion of job B and the dummy job. Since the dummy job is completed as soon as A is completed, event 3 in essence marks the completion of jobs A and B.

5.11 Rules for Network Construction :

The following are the primary rules for constructing AOA (Activity on Arrow) diagram.

1. The starting event and ending event of an activity are called tail event and head event, respectively.
2. The network should have a unique staring node (tail event).
3. The network should have a unique completion node (head event).
4. No activity should be represented by more than one are in the network.
5. No two activities should have the same starting node and the same ending node.
6. Dummy activity is an imaginary activity indicting precedence relationship only. Duration of a dummy activity is zero.

Example 5.6 Draw an arrow diagram showing the following relationships:

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Immediate predecessor	-	-	-	A,B	B,C	A,B	C	D,E,F	D	G	G	H,J	K	I,L

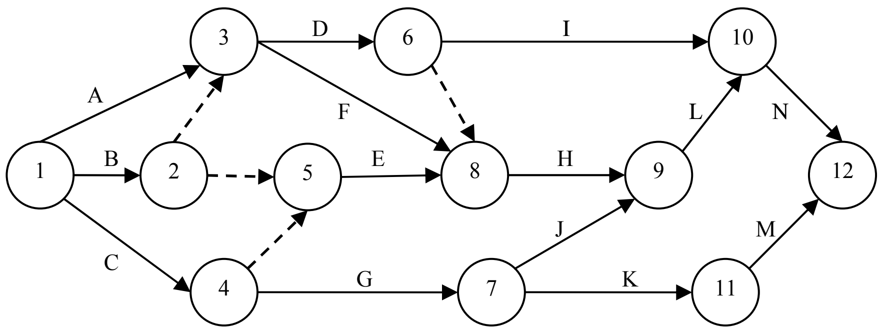


Fig 5.7

The use of the dummy 2-5 is very important here. It is necessary here because if it is eliminated, node 5 becomes the ending node of activity B and the initial node of activity E, implying that D and F require all A, B and C to be completed before their start, which is not the case. Inclusion of this activity thus enables us to present the precedence relationships in a correct manner.

5.12 Finding the Critical Path :

After the project network plan is completed and activity times are known, we consider the questions how long the project would take to complete and when the activities may be scheduled. This can be answered by finding out the critical path of the network. For this we require an arrow diagram and the time duration of the various activities. These computations involve a *forward* and a *backward* pass through the network. The forward pass calculations yield the *earliest start and the earliest finish times* for each activity, while the backward pass calculations render the *latest allowable start and the latest finish times* for each activity. We shall demonstrate the calculation of earliest start, earliest start, earliest finish, latest start and latest finish times of various activities of a project with the help of the following examples.

Example 5.7 Consider the following information on the activities required for a project.

Activity	:A	B	C	D	E	F	G	H	I	J	K	L
Immediate Predecessors	-	-	-	A	A	E	B	B	D,F	C	H,J	G,I,K
Duration :	2	2	2	3	4	0	7	6	4	10	3	4

Finding the critical path : To estimate how long the project will require, we will have to determine the critical path of this network. Since the work described by all the paths must be done before the project is considered complete, we must find that path that requires the most work, the longest path through the network; this is called the ***critical path***. If we want to reduce the time for the project, we will have to shorten the critical path; that is we will have to reduce the time of one or more activities on that path – but first we have to find it.

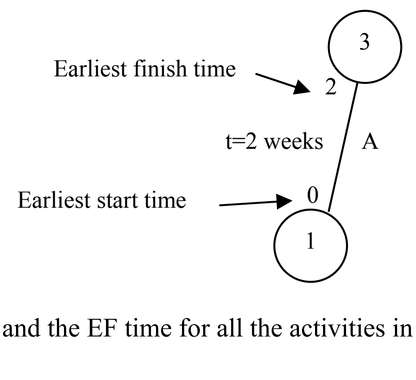
When the network is larger, it is very tedious, often impossible, to find the critical path by listing all the paths and picking the longest one. We need a more organized method. For this purpose, to begin with, a value of 0 (zero) is assigned to the initial event of the project. Thus, each of the activities initiated from the starting node of the network are assumed to start at time 0 (the beginning of the day 1, say). The earliest finish time for each activity is obtained by adding the time duration of the activity to its earliest start time.

We start at node 1 with a starting time we define as zero; we then compute an earliest start time and an earliest finish time for each activity in the network. Look at activity A with an expected time of 2 weeks:

To find the earliest finish time for any activity, we use this formula:

Earliest finish time = earliest start time + expected time

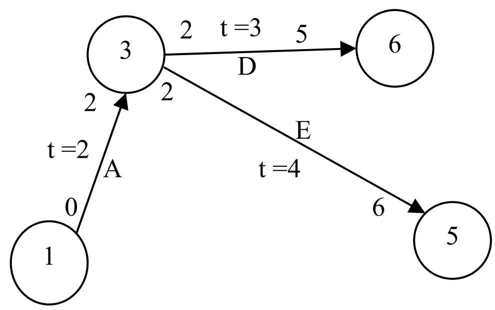
$$EF \quad = \quad ES \quad + \quad t$$



Now we must find the ES time and the EF time for all the activities in the network.

The earliest-start-time rule. ***"****Since no activity can begin until all its predecessor activities are complete, the earliest start time for an activity leaving any node is equal to the largest earliest finish time of all activities entering that same node*."

Look at the first few activities in the network.



In this instance, the earliest start time for activities D and E is 2, the earliest finish time for activity A. Using this procedure we make what is called a *FORWARD PASS* through the network to get all the ES and EF times as shown in the following figure 15.8 shown. We can see right away from the earliest finish time for activity L that it is going to take 19 weeks to finish this project, and that is only if all the activities run on schedule.

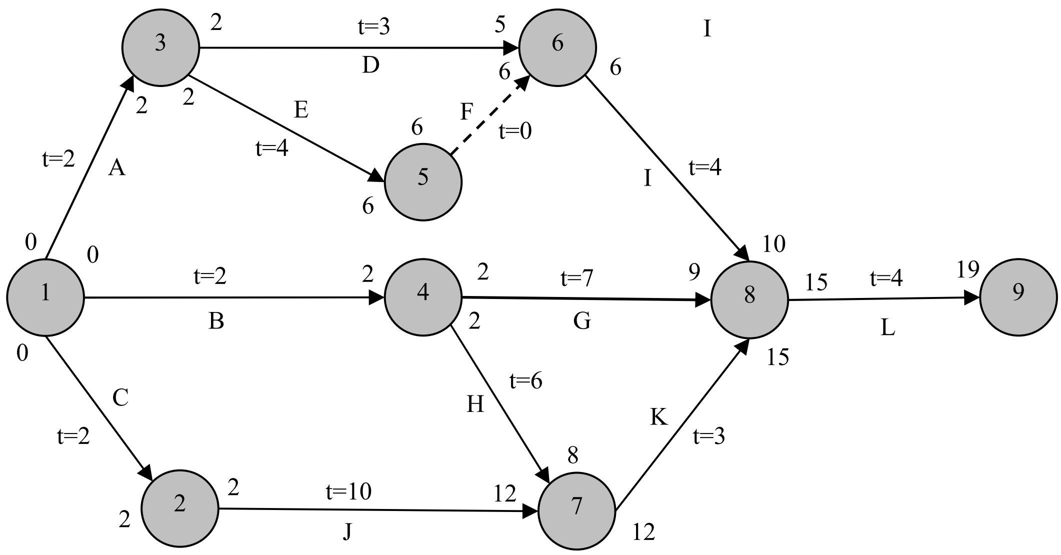


Fig 5.8

The second step in finding the critical path is to compute a latest start time and latest finish time for each activity. This is done by using what is called a *BACKWARD PASS*; that is, we begin at the completion point, node 9, and – using a latest finish time of 19 weeks for that activity (which we found in our forward pass method) – compute the latest finish time and latest start time for every activity. What is latest finish time? It is simply the latest time at which an activity can be completed without extending the completion time of the network. In the same sense, the latest start time is the latest time at which an activity can begin without extending the completion time on the project. In a more formal sense, the latest start time can be computed with

Latest start time = latest finish time - expected time

LS = LF - t

For example, given the latest finish time for activity L of 19 weeks, then

LS (for activity L) = 19 – 4 = 15

The latest-finish-time rule. **"***The latest finish time for an activity entering any node is equal to the smallest latest start time for all activities leaving that same node*."

Look at node 4 in following figure 5.9. The latest finish time for activity B entering that node is 6, the smallest start time for the two activities leaving node 4.

In figure we have shown the LS and LF times for all the activities in the network.

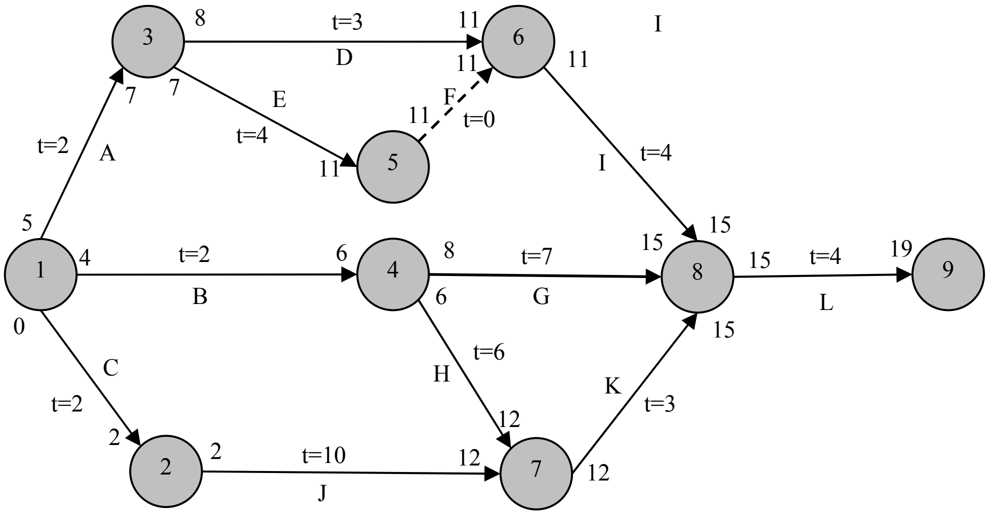


Fig 5.9

Now by comparing the *earliest start time* with the *latest start time* for any activity (that is, by looking at when it can be started compared with when it *must* be started), we see how much free time, or slack, that activity has. *SLACK* is the length of time we can delay an activity without interfering with the project competition. We can also determine slack for any activity by comparing its *earliest finish time* with its *latest finish time*. Look at activity A on the network in the figures 5.8 and 5.9.

LF - EF for activity A = 7 - 2 = 5

LS - ES for activity A = 5 - 0 = 5

The formal statement of these two methods is

Slack = LF - EF or LS - ES

In the following table 5.2, we have shown LF, EF, LS, ES, and slack for all the activities in the network.

Activity	Earliest Start ES	Latest Start LS	Earliest Finish EF	Latest Finish LF	Slack (LS-ES) or (LF-EF)	Activity on critical path
A	0	5	2	7	5	
B	0	4	2	6	4	
C	0	0	2	2	**0**	YES
D	2	8	5	11	6	
E	2	7	6	11	5	
F	Dummy Activity					
G	2	8	9	15	6	
H	2	6	8	12	4	
I	6	11	10	15	5	
J	2	2	12	12	**0**	YES
K	12	12	15	15	**0**	YES
L	15	15	19	19	**0**	YES

Table 5.2

Those activities without any slack are C, J, K, and L. None of these can be delayed without delaying the whole project. Thus, *the critical path for the our project is* C-J-K-L. We will have to watch these four activities especially closely; delay in any one of them will cause a delay in the project completion. Delays in other activities (A, B, D, E, G, H and I) will not affect on-time project completion (19 weeks) unless the delay is greater than the slack time an activity has. For example, it is all right for activity G to fall 6 weeks behind schedule because it has 6 weeks of slack; but if it falls more than 6 weeks behind schedule, it will delay competition of the project.

It can be observed that there may be more than one critical path in a given network. In case of multiple critical paths, all activities on these paths would be critical.

We can also construct the network directly as follows involving only the Early Start and Late Start calculation for events at nodes directly at the node as follows. [ES, LS] will be written at the node.

For the activities emanating from a given event, the ES time would be given by the earliest time of it and the LS time by the latest time of it. In the forward pass, a 0 would be taken as the earliest time for the initial event of the project and then for each subsequent event, the earliest time would be taken as the latest of the EF times of the activities concluding on that event. See the [ES,] entries in the following figure 5.10.

Similarly, the terminal event of the project would be assigned the latest time equal to its earliest time (or other time if it is given and desired). Then, rolling back, the events are assigned the latest times. If only one activity starts from the node representing a given event, then the latest time for the event is taken to be the difference between the latest time of the head event of this activity and the activity duration time. In case, however, more than one activity starts from this node, then the minimum of such differences, as mentioned above, would be taken as the latest time for the event. See the [ES, LS] entries in the following figure.

This method is to facilitate us in finding the critical path by calculating the slack at a node by directly computing LS *minus* ES.

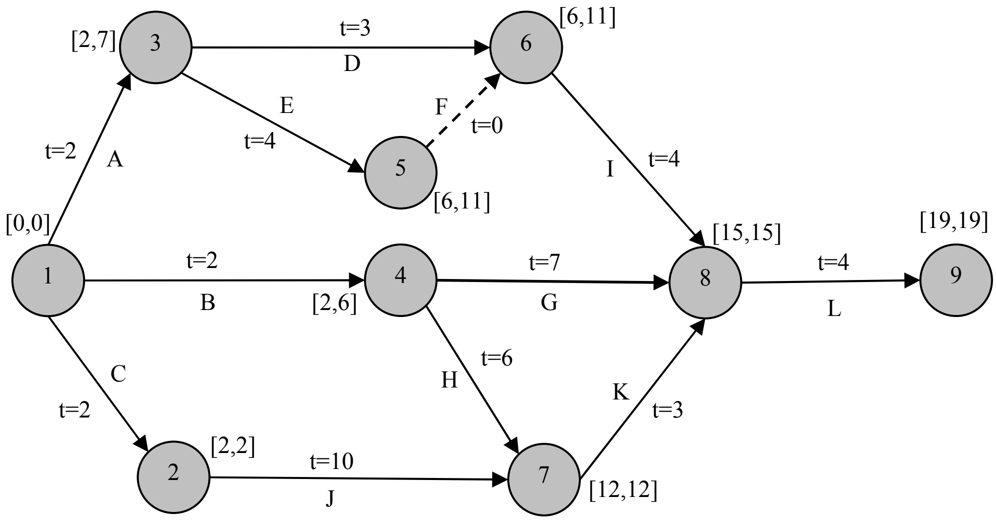


Fig 5.10

An alternative to depict the earliest and the latest timings for each of the activities is shown in the following example. Notice that each circle representing an event is divided into three parts: containing the event number, its earliest start time and its latest start time, as shown in the index.

Example 5.8

Assume the following network (fig. 15.11) has been drawn and the activity times estimated in days.

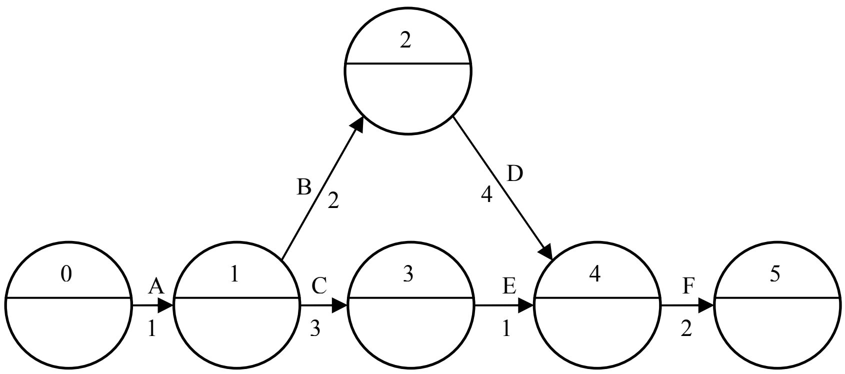


Fig 5.11

The ES times for the activities can be inserted as follows.

The ES of a head event is obtained by adding onto the ES of the tail event the linking activity duration starting from Event 0, time 0 and working forward through the network.

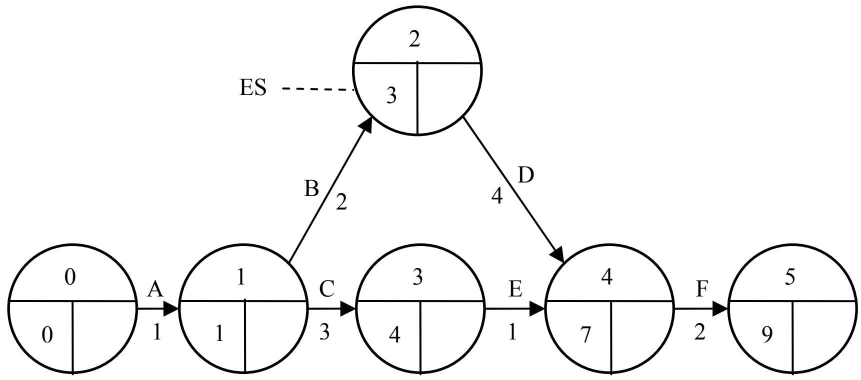


Fig 5.12

Where two or more routes arrive at an event the longest route time must be taken, e.g. Activity F depends on completion on D and E. E is completed by day 5 and D is not complete until day 7. Therefore F cannot start before day 7.

The ES in finish event No.5 is the project duration and is the shortest time in which the whole project can be completed.

The LS times are inserted as follows:

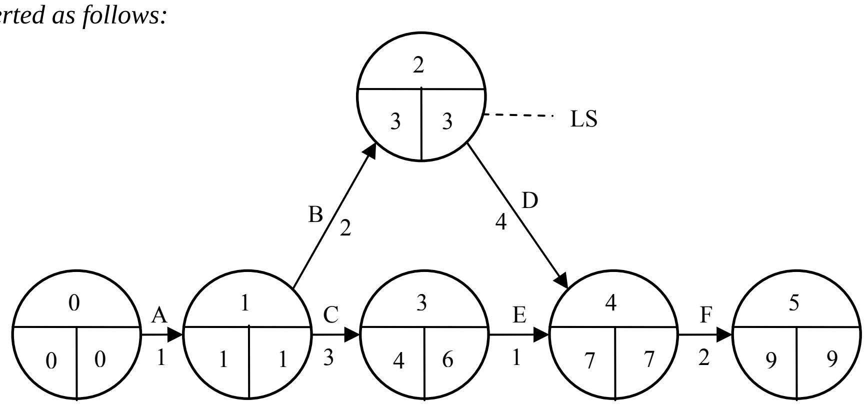


Fig 5.13

Starting at the finish event No.5, insert the LS (i.e. day 9) and work backwards through the network deducting each activity duration from the previously calculated LS.

Where the tails of activates B and C join event No.1, the LS for C is day 3 and the LS for B is day 1. The lowest number is taken as the LS for Event No.1 because if event No.1 occurred at day 3 then activities B and D could not be completed by day 7 as required and the project would be delayed.

***Finding the critical path* :** The above figure shows that one path through the network (A, B, D, F) has ES's and LS's which are identical. This is the critical path which it should be noted is the chain of activities which has the longest duration. The critical path can be indicated on the network either by a heavy line or different color or by two small transverse lines across the arrows along the path thus:

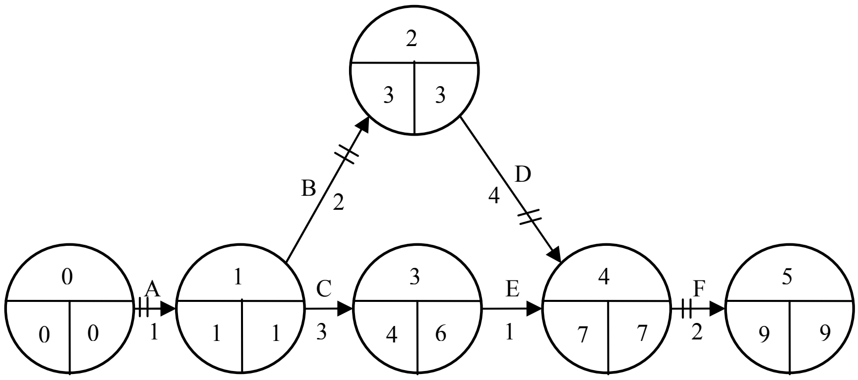
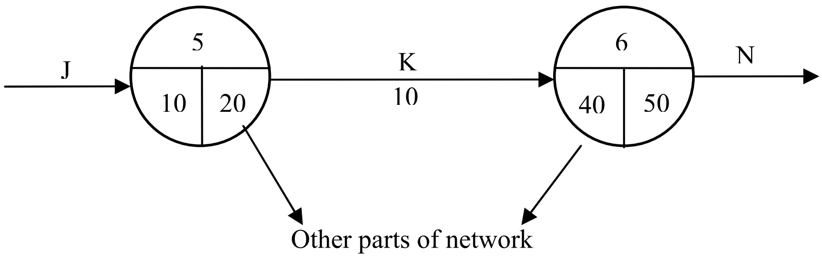


Fig 5.14

Critical path implications : The activities along the critical path are vital activities which must be completed by their ES/LS otherwise the project will be delayed. The non critical activities (in the example above, C and E) have spare time or float available i.e. C and / or E could take up to an additional 2 days in total without delaying the project duration. If it is required to reduce the overall project duration then the time of one or more of the activities on the critical path must be reduced perhaps by using more labor, or more or better equipment or some other method of reducing job times.

Floats: *Total, Independent and Free*

Float or spare time can only be associated with activities which are non-critical. By definition, activities on the critical path cannot have float. There are three types of float, Total Float, Free Float and Independent Float. To illustrate these types of float, part of a network will be used together with a bar diagram of the timings thus:



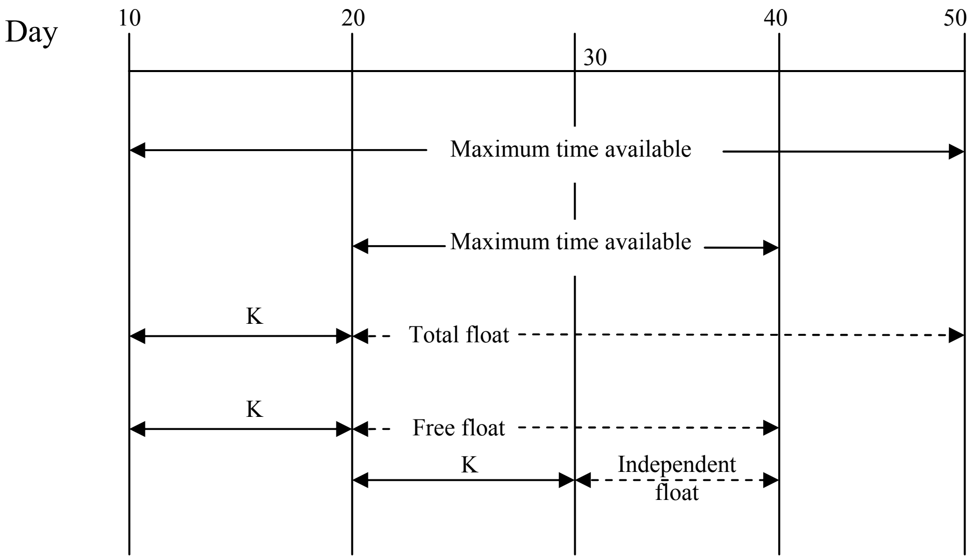


Fig 15.16

(a) Total float : This is the amount of time a path of activities could be delayed without affecting the overall project duration. (For simplicity the path in this example consists of one activity only i.e. Activity K).

Total Float = Latest Finish time – Earliest Start time – Activity Duration

Total Float = 50 -10 -10

= 30 days

(b) **Free float** : This is the amount of time an activity can be delayed without affecting the commencement of a subsequent activity at its earliest start time, but may affect float of a previous activity.

Free Float = Earliest Finish time – Earliest Start time – Activity Duration

Free Float = 40 -10 – 10

= 20 days

(c) **Independent float :** This is the amount of time an activity can be delayed when all preceding activities are completed as late as possible and all succeeding activities completed as early as possible. Independent float therefore does not affect the float of either preceding or subsequent activities.

Independent float = Earliest Finish time – Latest Start time – Activity Duration

Independent float = 40 -20 -10

= 10 days

Notes:

(a) The most important type of float is Total Float because it is involved with the overall project duration. On occasions the term 'Float' is used without qualification. In such cases assume that Total Float is required.

(b) The total float can be calculated separately for each activity but it is often useful to find the total float over paths of non-critical activities between critical events. For example in the fig. 5.16 of the previous example, the only non-critical path of activities is C, E for which the following calculation can be made:

Non-critical path	Time required	Time available	Total float over path
C, E	3 + 1 = 4 days	7-1 = 6 days	= 2 days

If some of the 'path float' is used up on one of the activities in a path it reduces the leeway available to other activities in the path.

Example 5.9 A simple network example is given.

Activity	Preceding activity	Duration
A	-	9
B	-	3
C	A	8
D	A	2
E	A	3
F	C	2
G	C	6
H	C	1
J	B, D	4
K	F, J	1
L	E,H,G,K	2
M	E,H	3
N	L,M	4

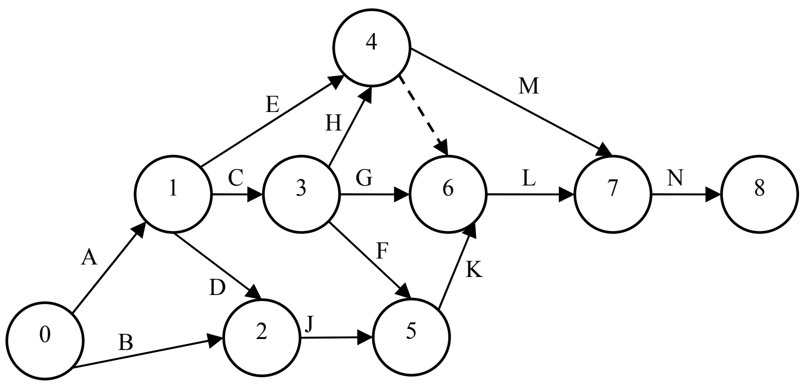


Fig 5.17

A dummy (4-6) was necessary because of the preceding activity requirements of activity L. If activities E, H had not been specified as preceding activity L, the dummy would not have been necessary.

The network is shown in the normal manner in the following figure from which it will be seen that the critical path is: A- C- G- L- N with a duration of 29 days.

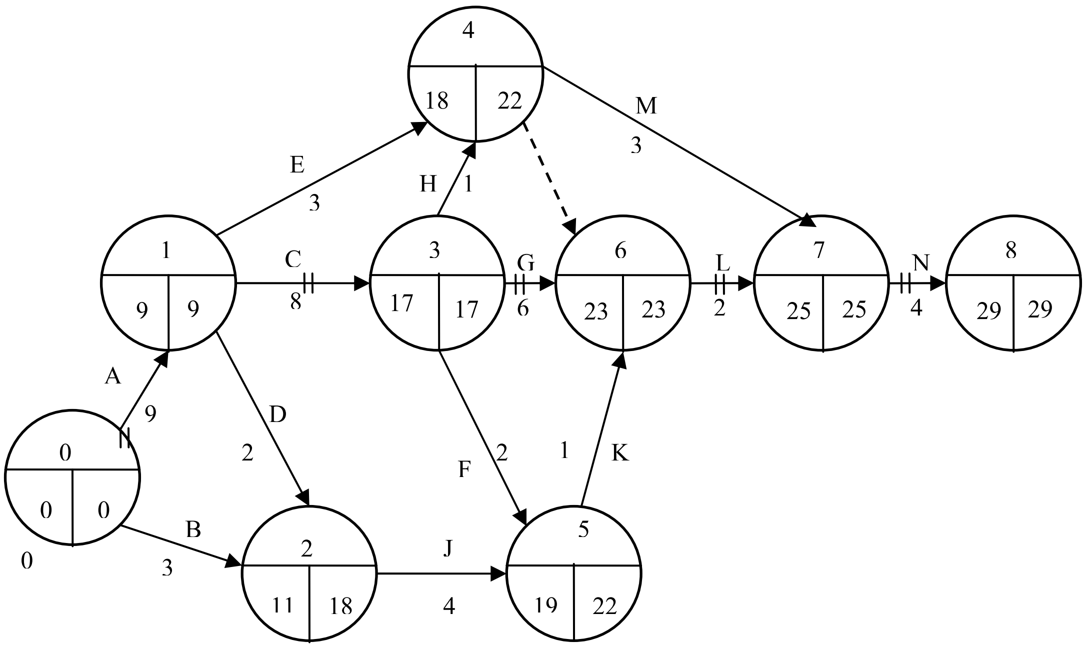


Fig 5.18

***The float calculations*:**

Activity	ES	LS	EF	LF	D	Total float (LF-ES-D)	Free float (EF-ES-D)	Independent float (EF-LS-D)
A	0	0	9	9	9	-	-	-
B	0	0	11	18	3	15	8	8
C	9	9	17	17	8	-	-	-
D	9	9	11	18	2	7	-	-
E	9	9	18	22	3	10	6	6
F	17	17	19	22	2	3	-	-
G	17	17	23	23	6	-	-	-
H	17	17	18	22	1	4	-	-
J	11	18	19	22	4	7	4	-
K	19	22	23	23	1	3	1	-
L	23	23	25	25	2	-	-	-
M	18	22	25	25	3	4	4	-
N	25	25	29	29	4	-	-	-

The total float on the non critical paths can also be calculated:

Non-critical path	Time required	Time available	Total float over path
B, J, K	8	23	15
D, J, K	7	14	7
F, K	3	6	3
E, M	6	16	10
H, M	4	8	4
E, dummy	3	14	9
H, dummy	1	6	5

Now, we see one more method for calculating the critical path and the floats.

Example 5.10 Consider the table summarizing the details of a project involving 14 activities .

Activity	Immediate Predecessors(s)	Duration (months)
A	-	2
B	-	6
C	-	4
D	B	3
E	A	6
F	A	8
G	B	3
H	C, D	7
I	C, D	2
J	E	5
K	F,G, H	4
L	F,G, H	3
M	I	13
N	J, K	7

The network is shown in fig. 5.19.

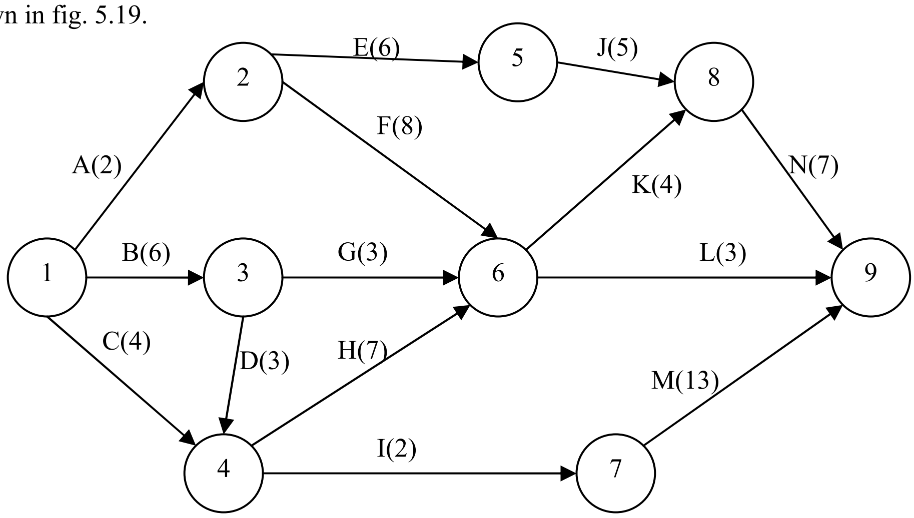


Fig 5.19

Compute the earliest start times and the earliest finish times and the late start and late finish times for each activities, the critical path and the floats associated with each activity. Check your answers with the following figure and the table.

The values for all nodes are computed and summarized in the figure 5.20 and written in the format [ES, LS] .

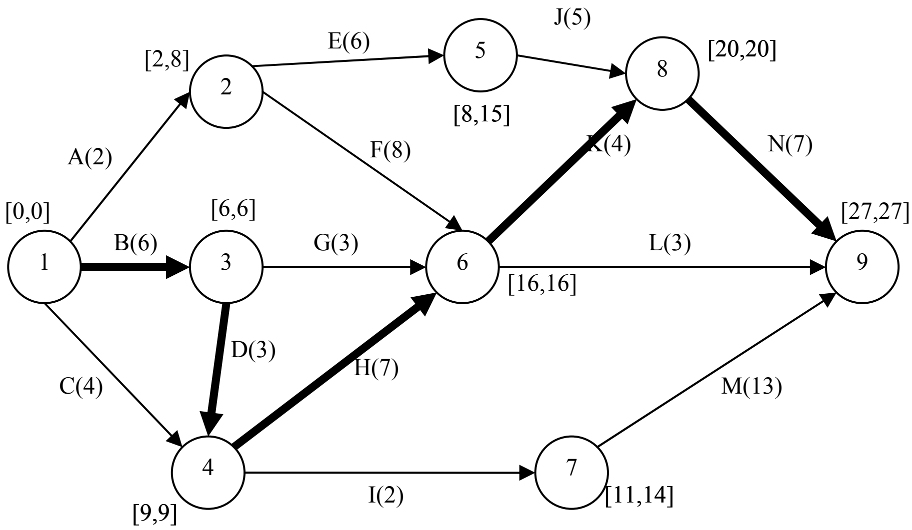


Fig 5.20

Determination of latest finish times

The values for all nodes are summarized in the above figure besides the ES values

The critical activities are identified and are shown in the same figure with thick lines on them. The corresponding critical path is 1-3-4-6-8-9 (B-D-H-K-N). The project completion time is 27 months.

Total Floats:

The calculation of total floats and free floats of the activities are summarized in the following table 5.3.

Activity (i,j)	**Duration (D_{ij})**	**Total float (TF_{ij})**	**Free float (FF_{ij})**
1-2	2	6	0
1-3	6	0	0
1-4	4	5	5
2-5	6	7	0
2-6	8	6	6
3-4	3	0	0
3-6	3	7	7
4-6	7	0	0
4-7	2	3	0
5-8	5	7	7
6-8	4	0	0
6-9	3	8	8
7-9	13	3	3
8-9	7	0	0

Table 5.3

Any critical activity will have zero total float and zero free float. Based on this property also, one can determine the critical activities. From the above table one can check that the total floats and free floats for the activities (1,3), (3,4), (4,6), (6,8) and (8,9) are zero. Hence, they are critical activities. The corresponding critical path is 1-3-4-6-8-9 (B-D-H-K-N).

5.13 Project Evaluation and Review Technique (PERT) :

So far in our analysis the probability considerations in the management of a project were not included. CPM assumed that the job times are known but can be varied by changing the level of resources. However, in all the research and development projects, many activities are performed only once. Hence, no prior experience with similar activities is available. The management of such projects is done by PERT, which takes into account uncertainties in the completion times of the various activities.

For each activity in the project network, PERT assumes three times estimates on its completion time. They include (1) a most probable time denoted by m, (2) an optimistic time denoted by a, and (3) a pessimistic time denoted by b.

The most probable time is the time required to complete the activity under normal conditions. To include uncertainties, a range of variation in job time is provided by the optimistic and pessimistic times. The optimistic estimate is a good guess on the minimum time required when everything goes according to plan whereas the pessimistic estimate is a guess on the maximum time required under adverse conditions such as mechanical breakdowns, minor labor troubles, or shortage of or delays in delivery of material. It should be remarked here that the pessimistic estimate does not take into consideration unusual and prolonged delays or other catastrophes. Because both these estimates are only qualified guesses, the actual time for an activity could lie outside this range. (From a probabilistic view point we can only say that the probability of a job time falling outside this range is very small.)

Most PERT analysis assumes a Beta distribution for the job times as shown in fig. 5.21 where μ represents the average length of the job duration. The value of μ depends on how close the values of a and b are relative to m.

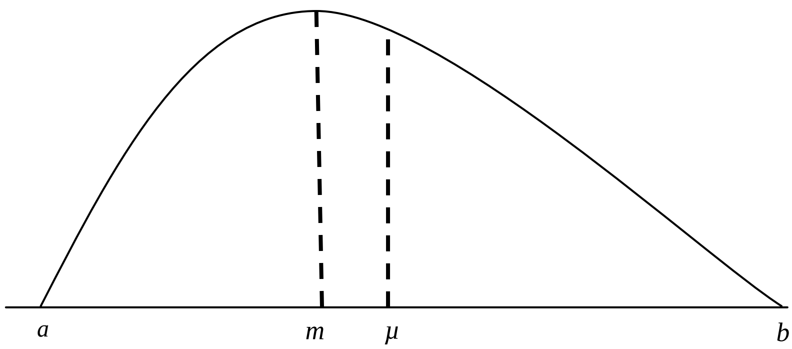


Fig 5.21 Beta distribution for job time.

The expected time to complete an activity is approximated as

$$\mu = \frac{a + 4m + b}{6} \qquad (1)$$

Since the actual time may vary from its mean value, we need the variance of the job time. For most unimodal distributions (with single peak values), the end values lie within three standard deviations from the mean value. Thus the spread of the distribution is equal to six times the standard deviation value (σ).

Thus $6\sigma = b - a$, or $\sigma = (b-a) / 6$. The variance of the job time equals.

$$\sigma^2 = \left(\frac{b-a}{6}\right)^2 \qquad (2)$$

With the three time estimates on all the jobs, PERT calculates the average time and the variance of each job using Eqs. 1 and 2. Treating the average times as the actual job times, the critical path is found. The duration of the project (T) is given by the sum of all the job times in the critical path. But the job times are random variables. Hence, the project duration T is also a random variable, and we can talk of the average length of the project and its variance.

The expected length of the project is the sum of all the average times of the jobs in the critical path. Similarly, the variance of the project duration is the sum of all the variances of the jobs in the critical path assuming that all the job times are independent.

Example 5.11 Assume that a simple project has the network shown in the following figure. The activity times are in weeks and three estimates have been given for each activity in the order *a, m, b*. The scheduled date for completion is week 19.

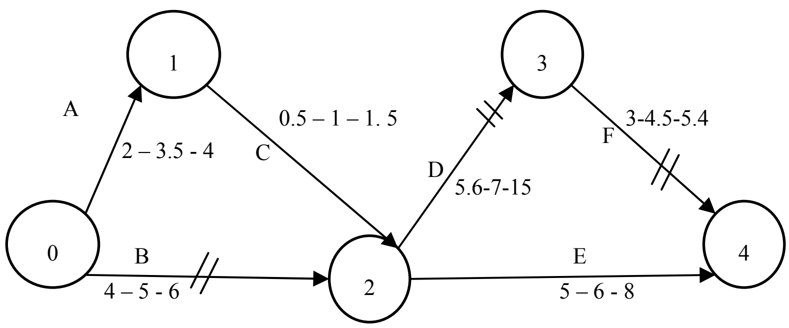


Fig 5.22

The expected durations can be found using what is known as the PERT formula:

$$\frac{\text{Optimistic time} + \text{Pessimistic time} + 4 \times \text{Most likely time}}{6}$$

Critical Path (B, D, F) expected duration

B = [4+6+4(5)]/6 = 5 weeks

D = [5.6+15+4(7)]/6 = 8.1 weeks

F = [3+5.4+4(4.5)]/6 = 4.4 weeks

Total 17.5 weeks

If the critical activities were to occur at their optimistic times, event 4 would be reached in 12.6 weeks but if the critical activities occurred at their pessimistic times, event 4 would be reached in 26.4 weeks. As these durations span the scheduled date of week 19 some estimate of the probability of achieving the schedule date must be calculated, as follows.

(a) Make an estimate of the Standard Deviation for each of the critical activities using

$$\frac{\text{Pessimistic time} - \text{Optimistic time}}{6}$$

i.e. Standard Deviation of Activity B = (6-4)/6 = 0.33

Activity D = (15-5.6)/6 = 1.57

Activity F = (5.4 -3)/6 = 0.4

(b) Find the standard deviation of event 4 by calculating the statistical sum (the square root of the sum of the squares) of the standard deviations of all activities on the critical path.

i.e. Standard Deviation of Event 4 = $\sqrt{0.33^2 + 1.57^2 + 0.4^2}$

= 1.65 weeks

(c) Find the number of event standard deviations that the scheduled date is away from the expected duration.

i.e. (19 – 17.5)/(1.65) = 0.91

(d) Look up this value (0.91) in a table of areas under the Normal Curve to find the probability. In this case the probability of achieving the scheduled date of week 19 is 82%.

Probability interpretation. If management consider that the probability of 82% is not high enough, efforts must be made to reduce the times or the spread of time of activities on the critical path. It is an inefficient use of resources to try to make the probability of reaching the scheduled date 100% or very close to 100%. In this case management may well accept the 18% chance of not achieving the schedule date as realistic.

Notes:

(a) The methods of calculating the Expected Duration and Standard Deviation as shown above cannot be taken as strictly mathematically valid but are probably accurate enough for most purposes. It is considered by some experts that the standard deviation, as calculated above, underestimates the 'true' standard deviation.

(b) When activity times have variations the critical path will often change as the variations occur.

Example 5.12 Consider a project consisting of nine jobs (A, B, :….., I) with the following precedence relations and time estimates :

Activity	Predecessors	Optimistic Time (a)	Most Probable Time (m)	Pessimistic Time (b)
A	-	2	5	8
B	A	6	9	12
C	A	6	7	8
D	B, C	1	4	7
E	A	8	8	8
F	D, E	5	14	17
G	C	3	12	21
H	F, G	3	6	9
I	H	5	8	11

First we compute the average time and the variance for each job. They are tabulated as follows in table 5.4:

Activity	Average Time	Standard Deviation	Variance
A	5	1	1
B	9	1	1
C	7	1/3	1/9
D	4	1	1
E	8	0	0
F	13	2	4
G	12	3	9
H	6	1	1
I	8	1	1

Table 5.4

Following figure 5.23 gives the project network, where the numbers on the arcs indicate the average job times. Using the average job times, the earliest and latest times of each event are calculated. The critical path is found as 1→2→4→5→6→7→8. The critical jobs are A,B,D,F,H and I.

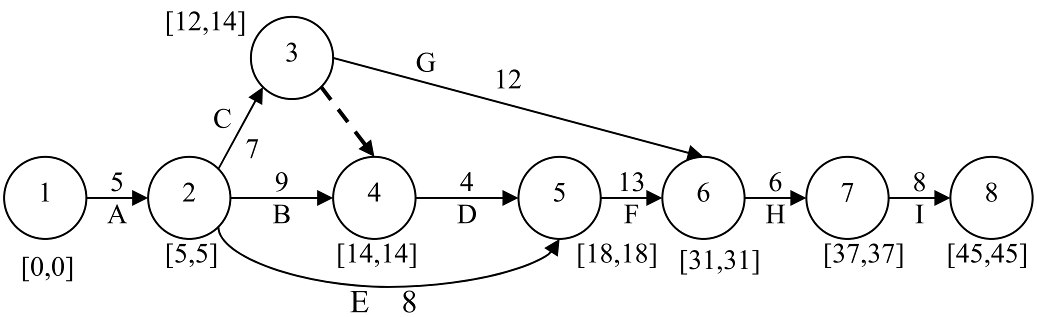


Fig 5.23

Let T denote the project duration. Then the expected length of the project is

E(T) = Sum of the expected times of jobs A, B, D, F, H and I = 5 + 9 + 4 + 13 + 6 + 8 = 45 days

The variance of the project duration is V(T) = Sum of the variance of jobs A, B, D, F, H and I = 1 + 1 + 1 + 4 + 1 + 1 = 9 and the standard deviation of the project duration is $\sigma(T) = 3$

Probabilities of Completing the Project

The project length T is the sum of all job times in the critical path. PERT assumes that all the job times are independent, and are identically distributed. Hence, by the *Central Limit Theorem,* T has a normal distribution with mean E(T), and variance V(T). The following figure exhibits a normal distribution with mean μ and variance σ^2.

In our example T is distributed normal with mean 45 and standard deviation 3. For any normal distribution, the probability that the random variable lies within one standard deviation from the mean is 0.68. Hence, there is a 68% chance that the project duration will be between 42 and 48 days. Similarly there is a 99.7% chance that T will lie within three standard deviations (i.e. between 36 and 54).

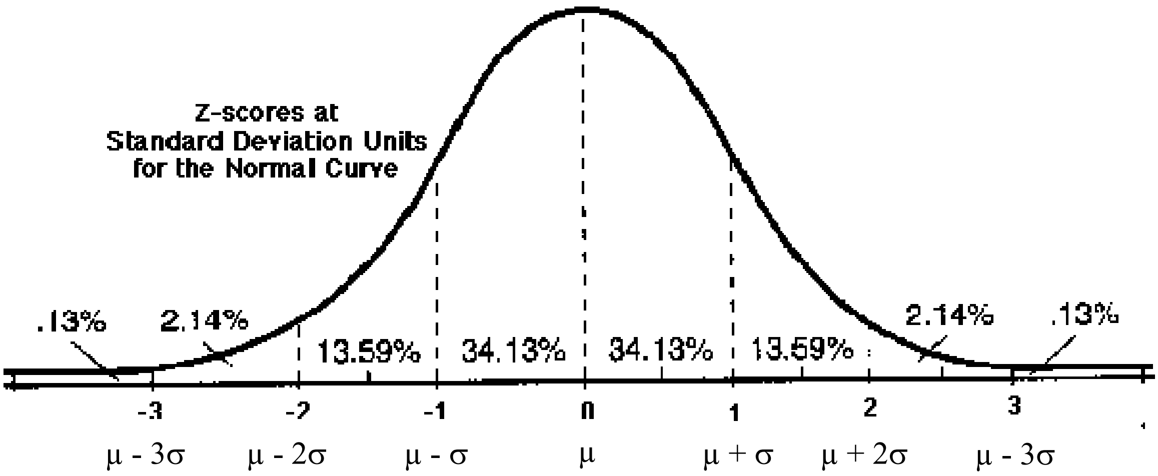


Fig 5.24

We can also calculate the probabilities of meeting specified project deadlines. For example, the management wants to know the probability of completing the project by 50 days. In other words, we have to compute Prob ($T \leq 50$) where $T \sim N(45, 3^2)$. This can be obtained from the tables of normal distribution; however, the tables are given for a standard normal only whose mean is 0 and standard deviation is 1.

From probability theory the random variable $Z=[T-E(T)]/\sigma(T)$ is distributed normally with mean 0 and standard deviation 1. Hence,

$$\text{Prob}(T \leq 50) = \text{Prob}\left(Z \leq \frac{50-45}{3}\right) = \text{Prob}(Z \leq 1.67) = 0.95$$

Thus there is a 95% chance that the project will be completed within 50 days.

Suppose we want to know the probability of completing the project 4 days sooner than expected. This means we have to compute.

$$\text{Prob } (T \leq 41) = \text{Prob } \left(Z \leq \frac{41\text{-}45}{3} \right) = \text{Prob } (Z \leq -1.33) = 0.09$$

Hence, there is only a small 9% chance that the project will be completed in 41 days.

Note: When multiple critical paths exist, the variance of each critical path may be different even though the expected values are the same. In such a circumstance, it is recommended that the largest variance of T be used for probability estimates.

Example 5.13 Consider the details of a project involving 11 activities.

Activity	Predecessor(s)	Duration (weeks)		
		a	M	B
A	-	6	7	8
B	-	1	2	9
C	-	1	4	7
D	A,	1	2	3
E	A,B	1	2	9
F	C	1	5	9
G	C	2	2	8
H	E,F	4	4	4
I	E,F	4	4	10
J	D,H	2	5	14
K	I,G	2	2	8

Find the expected completion of the project.

(a) What is the probability of completing the project on or before 25 weeks ?

(b) If the probability of completing the project is 0.84, find the expected project completion time.

Solution: The expected duration and variance of each activity are shown in the table.

The project network is shown in the figure 5.25.

The calculations of critical path based on expected durations are summarized in the following table 5.5. The critical path is A-Dummy-E-H-J and the corresponding project completion time is 20 weeks.

Activity	**Duration (weeks)**			**Mean duration**	**Variance**
	a	m	b		
A	6	7	8	7	0.11
B	1	2	9	3	1.78
C	1	4	7	4	1.00
D	1	2	3	2	0.11
E	1	2	9	3	1.78
F	1	5	9	5	1.78
G	2	2	8	3	1.00
H	4	4	4	4	0.00
I	4	4	10	5	1.00
J	2	5	14	6	4.00
K	2	2	8	3	1.00

Table 5.5

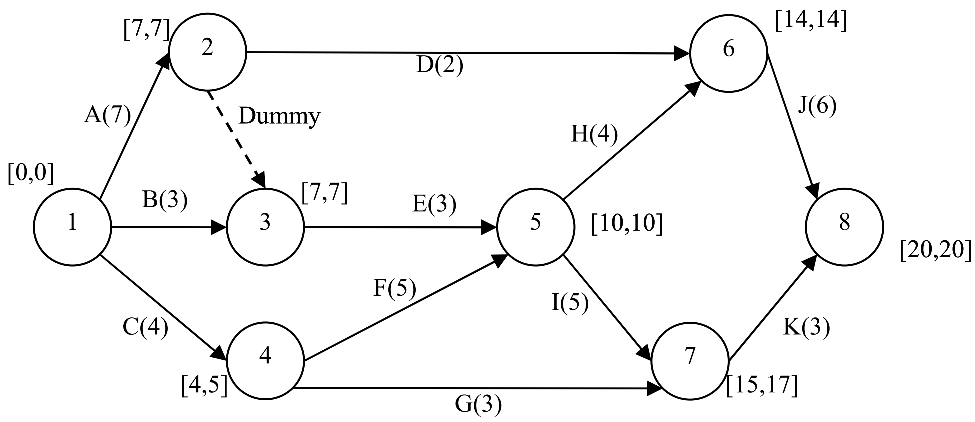


Fig 5.25

(a) The sum of the variances of all the activities on the critical path is:

$$0.11 + 1.78 + 0.00 + 4.00 = 5.89 \text{ weeks}$$

$$\sigma = \sqrt{5.89} = 2.43$$

$$P(x \le 25) = P\left(\frac{x-\mu}{\sigma} \le \frac{25-20}{2.43}\right) = P(z \le 2.06) = 0.9803$$

This value is obtained from standard normal table. Therefore, the probability of completing the project on or before 25 weeks is 0.9803.

(b) We also have P($x \le$ C) = 0.84.

Therefore, $$P\left(\frac{x-\mu}{\sigma} \le \frac{C-\mu}{\sigma}\right) = 0.84$$

$$P\left(z \le \frac{C-20}{2.43}\right) = 0.84$$

From the standard normal table, the value of z is 0.99, when the cumulative probability is 0.84.

Therefore, (C-20)/2.43 =0.99 or C = 22.4 weeks

The project will be completed in 22.41 weeks (approximately 23 weeks) if the probability of completing the project is 0.84.

Review Exercise

Q. Draw project network for the following activities and their predecessors

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M
Immediate predecessor	-	-	A	B	A,B	C,D	F,B	E,G	H,G	I,F	J,L	A	K

Q. Consider the following project activities and their duration

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
Imm. Pred	--	--	--	A	A	B	B	C	C	D	E	F	G	H	I	J,K,L	M,N,O
Duration (months)	4	8	5	4	5	7	4	8	3	6	5	4	12	7	10	5	8

(a) Construct the CPM network
(b) Determine the critical path
(c) Compute total floats and free floats for non critical activities

Q. Draw a network for the following activities and their predecessors

Activity	Immediate Predecessor	Activity	Immediate Predecessor
A	---	L	K
B	A	M	J
C	B	N	G
D	C	O	I,L
E	D	P	F
F	E	Q	O
G	F	R	P,Q
H	F	S	H,I
I	F	T	O,S
J	B	U	R,T
K	J		

Q. Draw the network for a project consisting of 16 jobs A,B,C,…..,M,N,O,P with the following job sequence

A,B,C,D → E,F,G
E,F,G → H
H → I
I → J,K,L,M,N
J,K,L,M,N → O
G,O → P

Q. The department of Mathematics of the University is holding a faculty development program. It has planned the following activities. Prepare a network diagram showing the interrelationships of various activities.

Activity	Description	Predecessors
A	Design conference meetings and theme	--
B	Design front cover of the conference proceedings	A
C	Prepare brochure and send request for papers	A
D	Complete list of distinguished speakers/guests	A
E	Finalize brochure and print it	C,D
F	Make travel arrangements for distinguished speakers/guests	D
G	Dispatch broachers	E
H	Receive papers for conference	G
I	Edit papers and assemble proceedings	F,H
J	Print proceedings	B,I

Q. Given the following information on a small project, draw a network based on this nformation.

Activity	A	B	C	D	E	F	G	H
Immediate predecessor	-	A	A	B,C	C	D	E	F,G

Q. Consider the following project activities and their predecessors and time estimates

Activity	A	B	C	D	E	F	G	H	I
Predecessor	--	--	A,B	A,B	B	D,E	C,F	D,E	G,H
Duration (days)	15	10	10	10	5	5	20	10	15

Determine the earliest completion time of the project, and identify the critical path

Q. Draw a network :

Activity	Immediate Predecessor	Activity	Immediate Predecessor
A	---	H	F
B	---	I	C,D,G,H
C	---	J	I
D	---	K	I
E	A	L	J,K
F	B	M	J,K
G	E	N	M

Q. For a small project of 12 activities, the details are given below. Draw the network and compute earliest occurrence time, latest occurrence time, critical activities and project completion time.

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Imm. Prede	--	--	--	B,C	A	C	E	E	D,F,H	E	I,J	G
Duration (days)	9	4	7	8	7	5	10	8	6	9	10	2

Q. The activities involved in a garment manufacturing company are listed with their time estimates as in the following table. Draw the network and carry out the critical path calculations

Activity	Description	Immediate predecessors	Duration (days)
A	Forecast sales volume	---	10
B	Study competitive market	---	7
C	Design item and facilities	A	5
D	Prepare production plan	C	3
E	Estimate cost of production	D	2
F	Set sales price	B,E	1
G	Prepare budget	F	14

Q. The information about various activities of a project and the precedence relationships are given below. Draw a network using as few dummies as possible

Activity	Immediate Predecessor	Activity	Immediate Predecessor
A	---	I	F,G,N
B	---	J	O,E,N
C	---	K	B,C,D
D	---	L	K
E	B,C,D	M	B,C,D
F	A,B,C,D	N	B,C,D
G	A,B,C,D	O	A,B,C,D
H	A,B,C,D	P	I,J,M,L

Q. Consider the following data regarding a project

Activity	Predecessors	Duration		
		Optimistic (a)	Most likely (m)	Pessimistic (b)
A	---	3	5	8
B	---	6	7	9
C	A	4	5	9
D	B	3	5	8
E	A	4	6	9
F	C,D	5	8	11
G	C,D,E	3	6	9
H	F	1	2	9

(a) Construct the project network

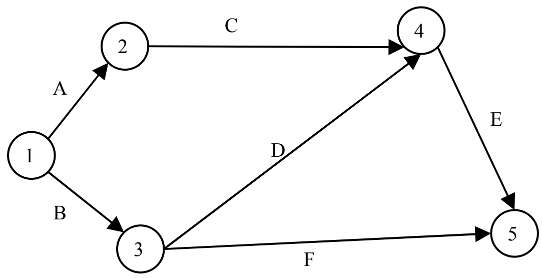
(b) Find the expected duration and the variance of the activity

(c) Find the critical path and the expected project completion time

(d) What is the probability of completing the project on or before 30 weeks?

(e) If the probability of completing the project is 0.9, find the expected project completion time.

Q. Consider the project network given below



The project executive has made estimates of the optimistic, most likely, and pessimistic time (in days) for completion of the various activities as follows

Activity	Duration		
	Optimistic (a)	Most likely (m)	Pessimistic (b)
A	2	5	14
B	9	12	15
C	5	14	17
D	2	5	8
E	6	9	12
F	8	17	20

(a) Find the critical path

(b) Determine the expected project completion time and its variance

(c) What is the probability that the project will be completed in 30 days?

Q. Schedule the following activities using CPM.

Activity	Immediate Predecessor	Time(weeks)
A	--	1
B	A	4
C	A	3
D	B	2
E	C,D	5
F	D	2
G	F	2
H	E,G	3

(a) Draw the network

(b) What is the critical path?

(c) How many weeks will it take to complete the project?

(d) Which activities have slack, and how much?

Q. The following represents a project that should be scheduled using CPM

Activity	Immediate Predecessor	Time(days)		
		a	m	b
A	--	1	3	5
B	--	1	2	3
C	A	1	2	3
D	A	2	3	4
E	B	3	4	11
F	C,D	3	4	5
G	D,E	1	4	6
H	F,G	2	4	5

(a) Draw the network

(b) What is the critical path?

(c) What is the expected project completion time?

(d) What is the probability of completing the project within 16 days?

Q. The home office billing department of a chain of department stores prepares monthly inventory reports for use by the store's purchasing agents. Given the following information, use the critical path method to determine

(a) How long the total process will take?

(b) Which jobs can be delayed without delaying the early start of any subsequent activity?

Job and its description	Immediate Predecessor	Time (weeks)
A: Start	--	0
B: Get computer printouts of customer purchases	A	10
C: Get stock records for the month	A	20
D: Reconcile purchase printouts and stock records	B,C	30
E: Total stock records by department	B,C	20
F: Determine reorder quantities for coming period	E	40
G: Prepare stock reports for purchasing agents	D,F	20
H: Finish	G	0

Q. Consider the following data regarding a project

Activity	Predecessors	Duration (weeks) (a)	(m)	(b)
A	---	4	4	10
B	---	1	2	9
C	---	2	5	14
D	A	1	4	7
E	A	1	2	3
F	A	1	5	9
G	B,C	1	2	9
H	C	4	4	4
I	D	2	2	8
J	E,G	6	7	8
K	F,H	2	2	8
L	F,H	5	5	5
M	I,J,K	1	2	9
N	L	6	7	8

(a) Construct the project network

(b) Find the expected duration and the variance of the activity

(c) Find the critical path and the expected project completion time

(d) What is the probability of completing the project on or before 35 weeks?

(e) If the probability of completing the project is 0.85, find the expected project completion time.

Q. Consider a project given below

Activity	Predecessors	Duration (a)	(m)	(b)
A	---	2	5	8
B	A	6	9	12
C	A	5	14	17
D	B	5	8	11
E	C,D	3	6	9
F	---	3	12	21
G	E,F	1	4	7

(a) Construct the project network

(b) Find the expected duration and the variance of the activity

(c) What is the expected length of the project, and its variance?

(d) What is the probability of completing the project

(1) 3 days earlier than expected

(2) No more than 5 days later than expected.

Q. Find the total and the free slack associated with each of the activities of the project

Activity	A	B	C	D	E	F	G	H	I	J
Imm. Prede	--	--	--	A,B	A,B	C	C	F,G	D	E,I
Duration (days)	11	8	6	3	7	7	4	16	5	9

Q. A project consists of the following activities:

Activity	A	B	C	D	E	F	G
Imm. Prede	--	--	A	B,C	B,C	D	E,F
Duration (weeks)	6	9	9	3	12	6	3

(a) Draw a network
(b) Compute ES,EF,LS and LF for each activity
(c) What is the project completion time? Which of the activities must be completed in time so that the project may be completed in time?
(d) If, for activity E, the immediate predecessors are not B and C, but instead D, how would, if at all, it affect the project duration?

Q. The data regarding a project is given below:

Activity	Predecessors	Duration (days)		
		(a)	(m)	(b)
A	---	2	2	2
B	---	1	3	7
C	A	4	7	8
D	A	3	5	7
E	B	2	6	9
F	B	5	9	11
G	C,D	3	6	8
H	E	2	6	9
I	C,D	3	5	8
J	G,H	1	3	4
K	F	4	8	11
L	J,K	2	5	7

Draw the PERT Network. Indicate the expected total slack for each activity and hence indicate the average critical path. Within how many days would you expect the project to be completed with 99% chance?

Q. Consider the following data regarding a project

Activity	Predecessors	Duration (weeks)		
		(a)	(m)	(b)
A	---	4	5	12
B	---	1	1.5	5
C	A	2	3	4
D	A	3	4	11
E	A	2	3	4
F	C	1.5	2	2.5
G	D	1.5	3	4.5
H	B,E	2.5	3.5	7.5
I	H	1.5	2	2.5
J	F,G,I	1	2	3

(a) Construct the project network
(b) Find the expected duration and the variance of the activity
(c) Find the critical path and the expected project completion time
(d) What is the probability of completing the project (1) in 20 weeks (2) in 15 weeks