Chapter 5

Sequential Decision Making

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Key words: Utility theory, decision trees, cutting of decision trees and sensitivity analysis.

Suggested readings:

1. Winkler R. L. and Hays W. L. (1975), **Statistics: Probability, Inference, and Decision** (2nd edition), Holt, Rinehart and Winston
2. Gupta P.K. and Mohan M. (1987), **Operations Research and Statistical Analysis**, Sultan Chand and Sons, Delhi.
3. Hillier F.S. and Lieberman G.J. (2005), **Introduction to Operations Research, (8th edition)**, Tata-McGraw Hill Publishing Company Limited.
4. Johnson R.D. and Bernard R.S. (1977), **Quantitative Techniques for Business Decisions**, Prentice hall of India Private Limited
5. Kemey J.G., Schleifer J.A. and Thompson G.L. (1968), **Finite Mathematics with Business Applications**, Prentice hall of India Private Limited.
6. Levin R.I. and Rubin D.S. (1998), **Statistics for Management**, Pearson Education Asia.
7. Levin R.I., Rubin D.S. and, Stinson J.P. (1986), **Quantitative Approaches to Management (6th edition)**, McGraw Hill Book company.
8. Raiffa H. and Schlaifer R. (1968), **Applied Decision Theory**, MIT Press.
9. Swarup K., Gupta P.K. and Mohan M. (2001), **Operations Research**, Sultan Chand and Sons, Delhi.

5.1 Introduction

In the earlier chapter, we have studied the importance of utility and expected monetary value. Also we have studied how decisions can be made as to maximize the utility and / or EMV. Now, consider again the following situation:

As we all know, oil is a scarce commodity and as such oil extraction is an area where many firms would like to venture into. There is a piece of land which is expected to have some oil as oil has been found in neighboring areas too. But some testing is needed to find out if any oil is there in the land or not. Testing is an expensive and time consuming program and there are 30% chances that testing would result in success. Even in case of success, the probability is 0.20 that the venture would be profitable which means that the amount of oil present would be 1, 00,000 barrels or more and 10% chances are that the venture would not be profitable. In fact the firm which owns the land has estimated the following states of nature with respective probabilities (without testing):

State of nature	**Probability**
No oil:	0.70
Less than 1, 00,000 barrels:	0.15
Less than 8, 00,000 barrels:	0.10
Less than 15, 00,000 barrels:	0.05

The firm has following options before it:

(I) **Do not test:**

(i) **Drill:** (a) If oil is found, whole profit would belong to the firm. In case of no oil, the firm would have to bear the whole loss.

(b) As the stakes are high that there will be no oil, in order to reduce risk, the firm may opt for a 50% partnership with another firm which would be applicable both in case of oil or no oil.

(ii) **Sell the drilling rights** for 1/5 of revenue if any. In this case, the firm will not have to bear the losses.

(iii) **Do not drill and sell the land** to another firm for Rs. 10, 00,000.

(II) **Test:** (i) If success (a) Drill

(b) **Do not drill and sell the land** to another firm for Rs. 50, 00,000.

(ii) If failure (a) Drill (testing may not be 100% accurate)

(c) **Do not drill and sell the land** to another firm for Rs. 5, 00,000.

In case tests are conducted, the probabilities of different states of nature are revised as follows:

(a) Testing successful

State of nature	**Probability**
No oil:	0.30
Less than 1, 00,000 barrels:	0.20
Less than 8, 00,000 barrels:	0.40
Less than 15, 00,000 barrels:	0.10

(a) Testing unsuccessful

State of nature	**Probability**
No oil:	0.80
Less than 1, 00,000 barrels:	0.10
Less than 8, 00,000 barrels:	0.08
Less than 15, 00,000 barrels:	0.02

What should be the decision of the firm? The profit of the firm per barrel of oil is Rs. 50 and the cost of testing is Rs. 5, 00,000. Cost of drilling is Rs. 10, 00,000.

In such cases, a single decision may not serve the purpose and decisions may be needed to be taken in sequence to solve the problem. First of all the firm has to decide whether to carry test or not. If it decides not to test whether it would proceed for drilling or not or whether it would sell the drilling rights. If the firm proceeds for drilling, it has to decide whether drilling has to be done in partnership or alone. If the firm decides to test, how should it proceed in case the tests are successful? Also it has to decide the course of action in case of failure of tests.

5.2 Decision Trees

A decision tree is a graphical representation of all the alternatives associated with a problem. Any problem, which can be solved with the help of decision trees, can be solved with the help of pay-off matrices also. But decision trees present a more comprehensive view of the problem.

Consider a system of three urns, which are identical in size, shape and color. First urn contains three red and one black ball; second urn contains two red and two black balls and the third urn contains one red and three black balls. An urn is chosen at random and two balls are drawn sequentially. We are interested in knowing the number of ways in which various combinations of red and black balls can be selected.

It is possible to write all the combinations in which a draw can result. However, we are interested in solving the problem pictorially.

The decision tree starts with the point where the first decision is to be taken. A decision point, also known as a decsion node, is represented by a rectangular box. From decision node, branches arise which indicate the various states of nature at that point. A possible state of nature is denoted by a circle and is known as a chance node. From chance nodes, again branches arise to indiacte further decisions which might be taken. The process goes till the last decision has been taken. Then moving sequentiqlly from beginning to end along a path, all the possible actions can be traced on a decision tree.

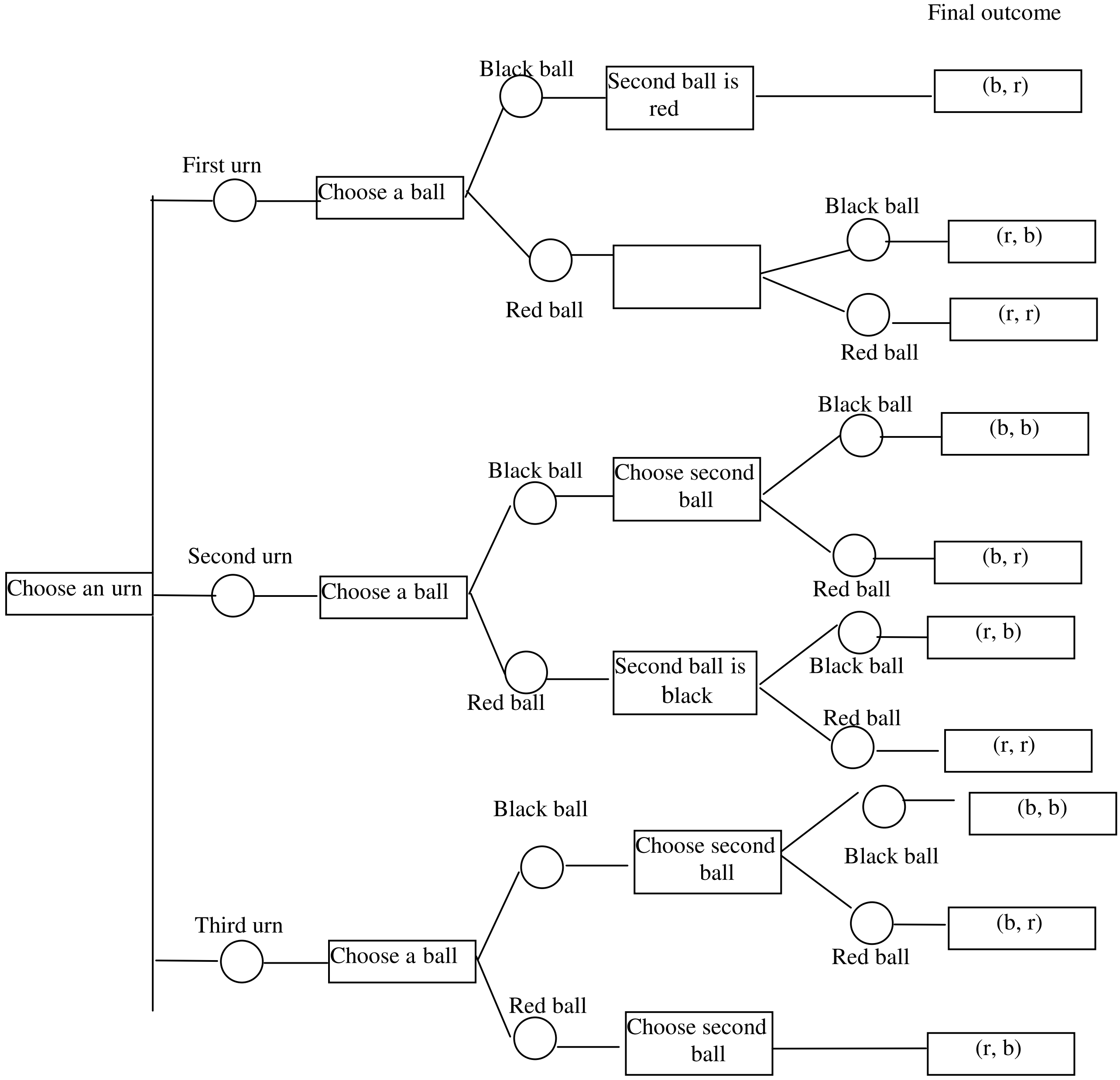

Fig. 5.1

5.3 Cutting of Decision trees

One way of evaluating a decision problem using tree approach is to evaluate the tree from the last branch. Then we move backwards to determine the optimal course of action. Now, we shall show, how thw decision trees can be used in calculating pay-offs and taking financial decisions. First of all we draw the decision tree of the problem considered at the beginning of the chapter and see how it can be cut to compute various pay-offs. Initially the company has two options, to test or not to test. Once the decision (on paper) about testing is made, the outcomes of both the decisions are weighed. However, the outcomes can not be weighed directly. For example, suppose the management is interested in knowing the profits if it decides in favor of testing. Then the options before the company depend upon whether the tesing would be successful or not. If yes, then the company has to decide between whether to carry our drilling or to sell the land. In case of drilling, the profits depend upon whether it finds any oil and if yes, how much of it? Similarly, if testing results in failure, then again the company has to decide between whether to carry our drilling or to sell the land. Again, in case of drilling, the profits depend upon whether it finds any oil and if yes, how much of it? If the company does not decide in favor of testing, then the options before the company are whether to drill or sell the rights of the land ot not to drill and sell the land. All these options are drawn sequentially and the final branches of the tree represent the outcomes (monetary) of all the options before the company.

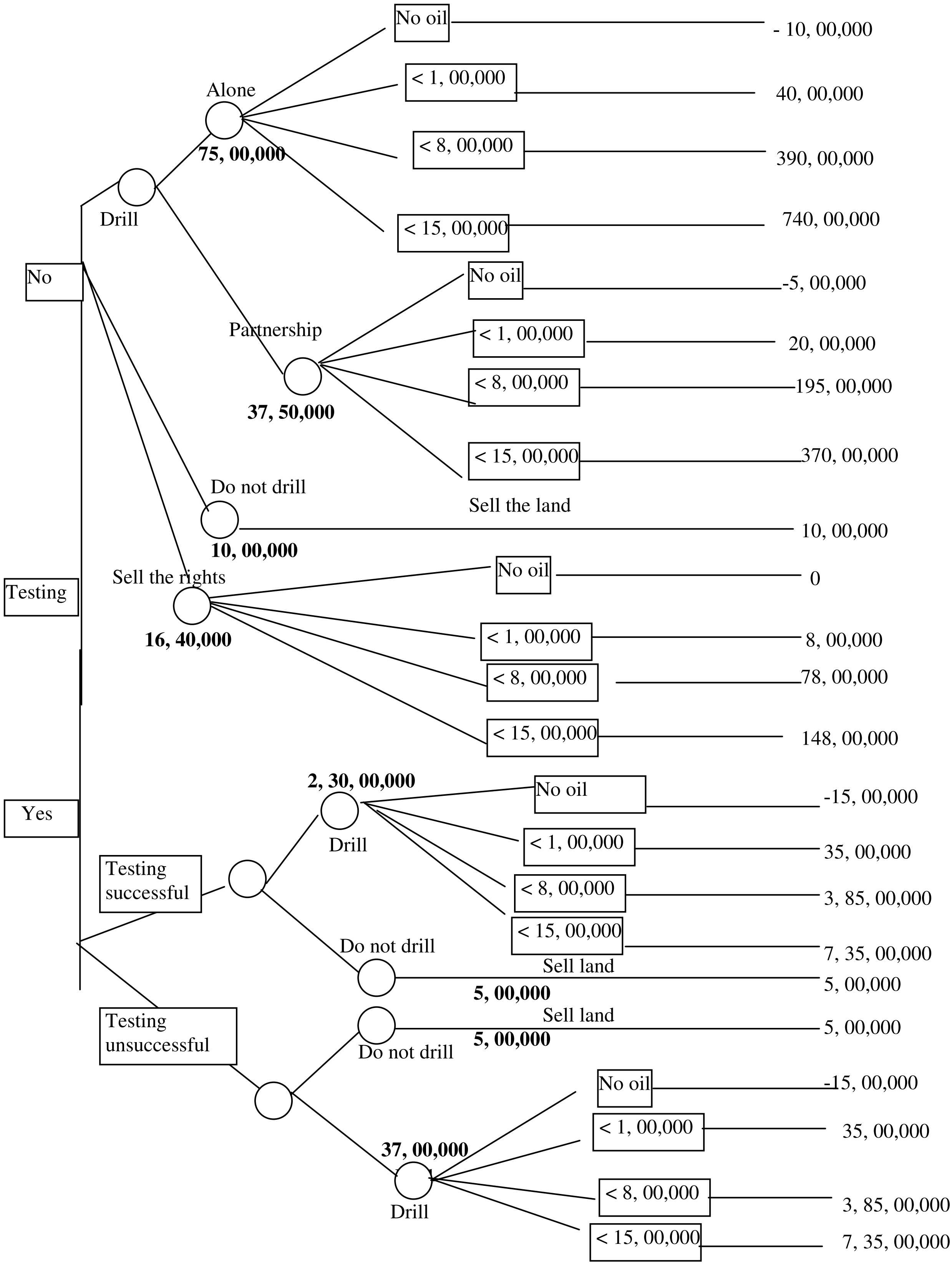
No oil
- 10, 00,000
< 1, 00,000
40, 00,000
Alone
< 8, 00,000
390, 00,000
75, 00,000
< 15, 00,000
740, 00,000
Drill
No
No oil
-5, 00,000
Partnership
< 1, 00,000
20, 00,000
< 8, 00,000
195, 00,000
37, 50,000
< 15, 00,000
370, 00,000
Do not drill
Sell the land
10, 00,000
10, 00,000
Sell the rights
No oil
0
Testing
16, 40,000
< 1, 00,000
8, 00,000
< 8, 00,000
78, 00,000
< 15, 00,000
148, 00,000
2, 30, 00,000
No oil
-15, 00,000
Yes
Drill
< 1, 00,000
35, 00,000
Testing successful
< 8, 00,000
3, 85, 00,000
< 15, 00,000
7, 35, 00,000
Do not drill
Sell land
5, 00,000
5, 00,000
Sell land
Testing unsuccessful
5, 00,000
5, 00,000
Do not drill
No oil
-15, 00,000
< 1, 00,000
35, 00,000
37, 00,000
< 8, 00,000
3, 85, 00,000
Drill
< 15, 00,000
7, 35, 00,000

Fig. 5.2

Now, we cut the branches of the tree.

The first point of sectioning is whether to test or not. If the company decides not to cut, then only the upper portion of the tree will be relevant.

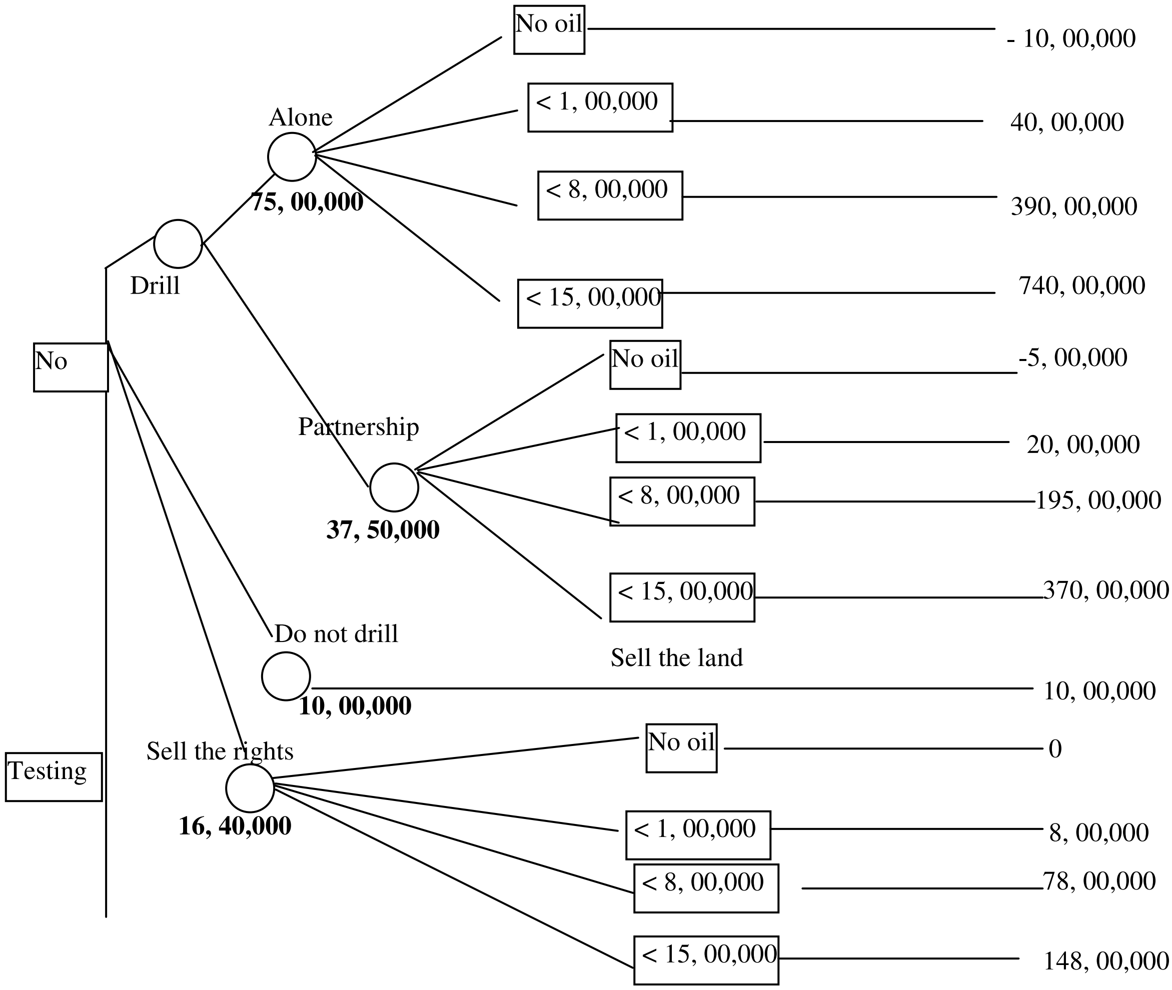

Fig. 5.3

Now, the options before the company are

(i) To drill: alone or in partenership

(ii) To sell the drilling rights: and

(iii) Not to drill and to sell the land.

Suppose the company decides in favor of drilling. Then the relevant part of the tree diagram is given as follows:

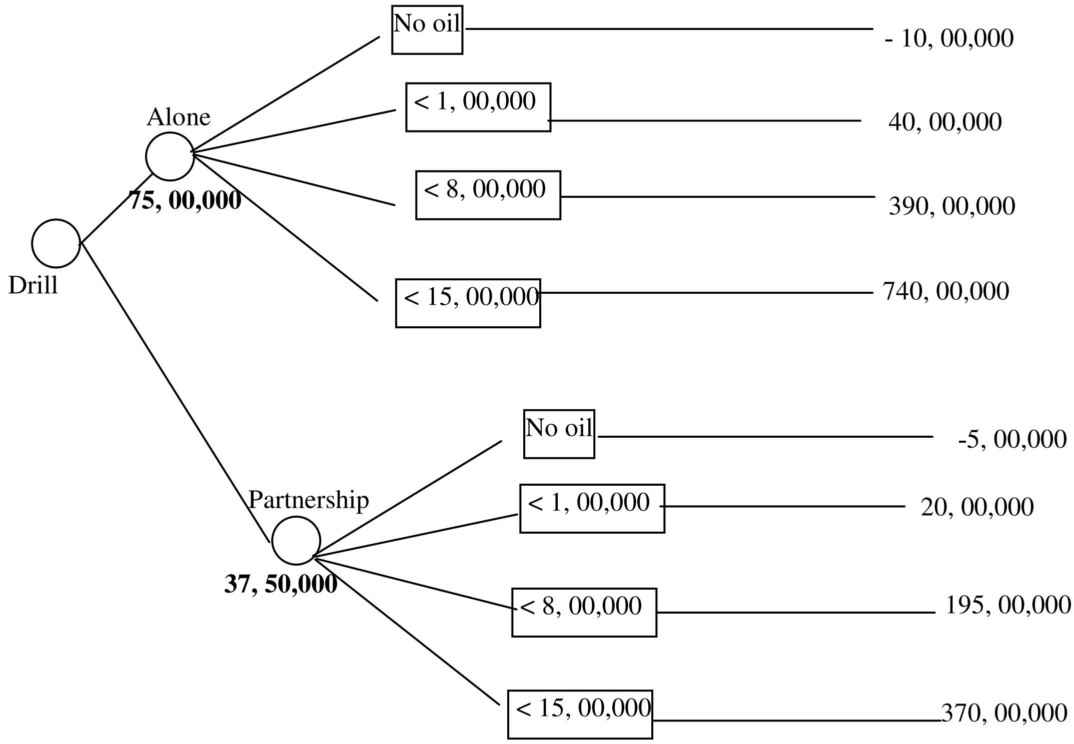

Fig. 5.4

If the firm decides to drill alone, then it has to calculate the pay-offs for the four states of nature and then to take a decision.

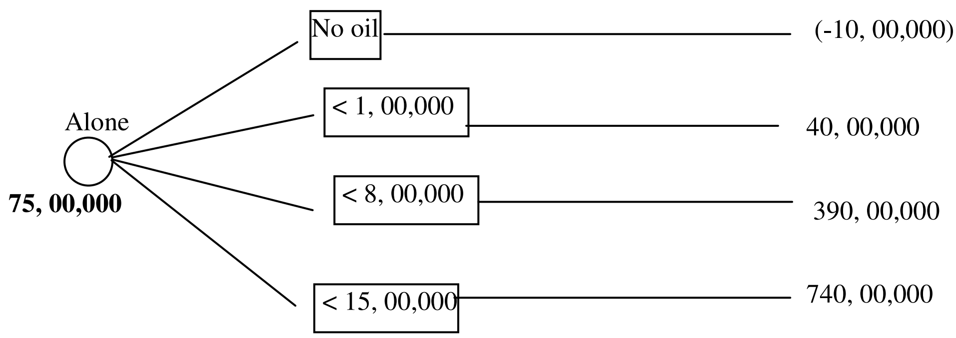

Fig. 5.5

Let us calculate the pay-offs of the firm with respect to these states of nature:

Table 5.1

State of nature	Probability	Pay-off (Rs.)
No oil	0.70	-10,00,000 (cost of digging)
Less than 1, 00,000 barrels:	0.15	(1,00,000)(50) - 10,00,000 = 40,00,000
Less than 8, 00,000 barrels:	0.10	3,90,00,000
Less than 15, 00,000 barrels	0.05	740,00,000

Expected pay-off of the decision =

(0.70)(-10,00,000)+(0.15)(40,00,000) +(0.10)(3,90,00,000) +(0.05)(740,00,000)

= Rs. 75, 00,000

If the firm decides to drill in partnership, then its profits/losses would be half of what these are if it decides to go alone. So we have

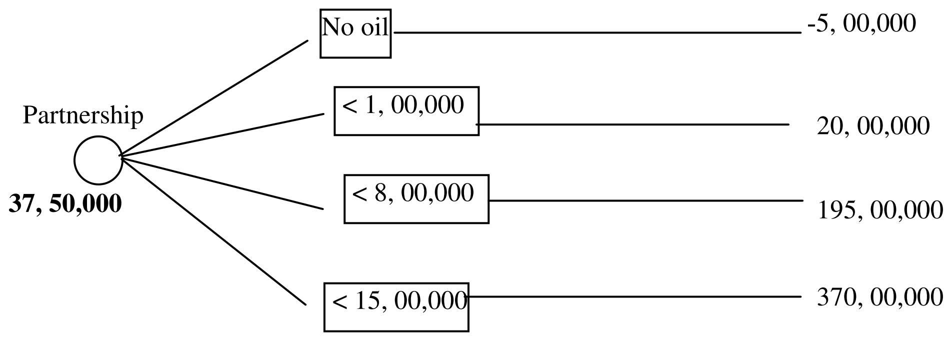

Fig. 5.6

If the firm decides to sell the rights, it would bear no losses but the profits would be 1/5 of what these were had it decided to drill. So we have

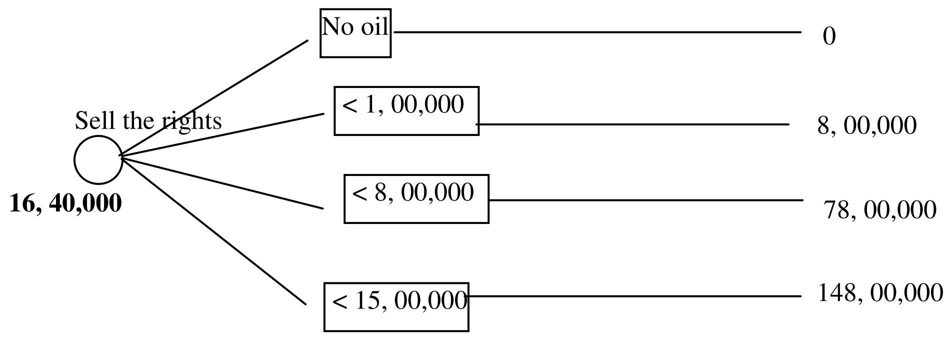

Fig 5.7

However, if the firm decides to sell the land, it would earn a profit of Rs. 10, 00,000.

Similarly, if the firm decides to go for testing, the tree can be cut at points if testing is successful or unsuccessful and decisions can be taken. The following pictures represent the sections of the tree.

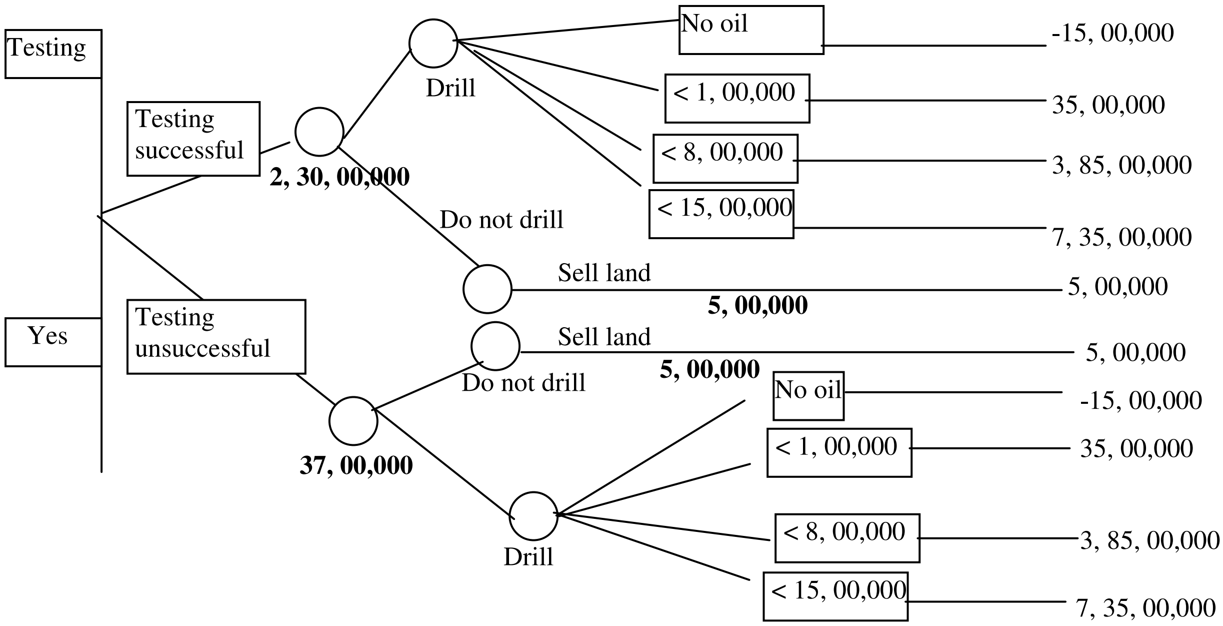

Fig. 5.8

(i) Testing successful

Table 5.2

State of nature	**Probability**	**Pay-off (Rs.)**
No oil	0.30	**-**15,00,000 (cost of testing and digging)
Less than 1, 00,000 barrels:	0.20	35,00,000
Less than 8, 00,000 barrels:	0.40	3,85,00,000
Less than 15, 00,000 barrels	0.10	7,35,00,000

Expected pay-off = Rs. 2, 30, 00,000

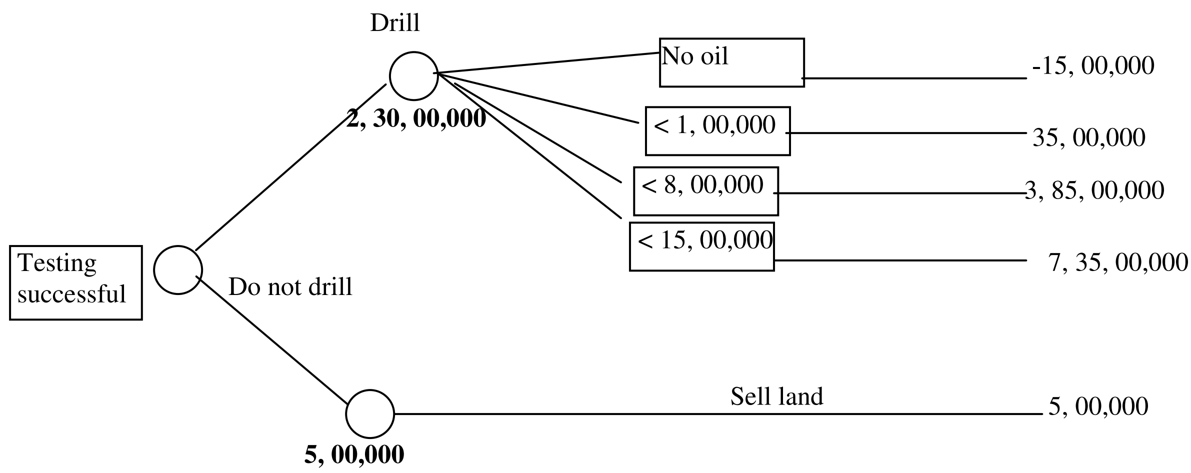

Fig. 5.9

(ii) Testing unsuccessful

Table 5.3

State of nature	**Probability**	**Pay-off (Rs.)**
No oil	0.80	-15,00,000
Less than 1, 00,000 barrels:	0.10	35,00,000
Less than 8, 00,000 barrels:	0.08	3,85,00,000
Less than 15, 00,000 barrels	0.02	7,35,00,000

Expected pay-off = Rs. 37, 00,000

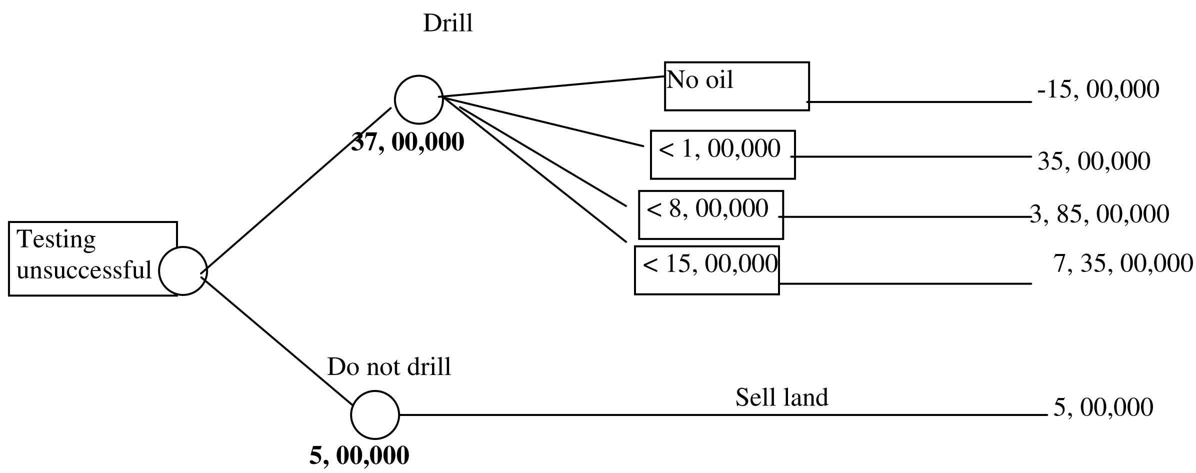

Fig. 5.10

Now, the simplified tree becomes

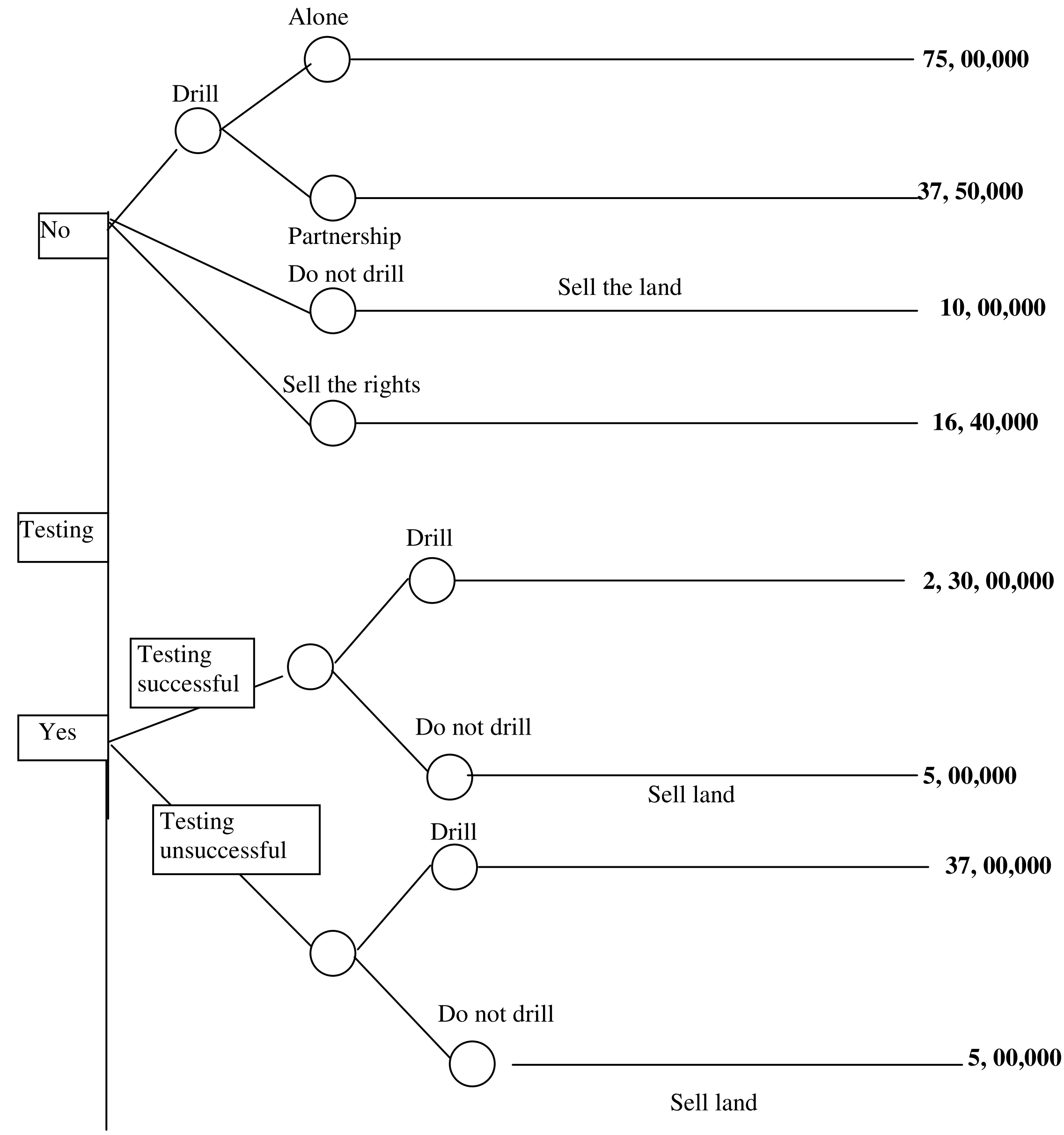

Fig. 5.11

Now, we can reach at the following conclusions:

(i) If the firm decides not to go for testing, then it should undertake the drilling alone as it would maximize its expected pay-off.

(ii) If the firm goes for testing, then in any case, it should undertake drilling.

5.4 Some more examples

Example 1: ABC Industries Ltd. Is considering a business expansion programme and has the following options before it:

(a) To manufacture a new product. For this option, the company would have to commission a new plant at a cost of Rs. 2,00,00,000. The life of this product will be 10 years.

(b) To increase the market share of the present product. For this the company would have to upgrade the present facilities; and to improve the advertising and marketing policies. This would cost the company Rs. 75,00,000 and the product would have a life of 6 years.

The CVP analysis has helped the management to reach the following conclusions:

(i) The demand of the products can be high (with probability 0.5), moderate (with probability 0.3) or low (with probability 0.2)

(ii) In case of high demand, the new product would fetch an annual profit of Rs. 40,00,000. However, if the demand is moderate, the annual profit will be in tunes of Rs. 25,00,000. In case of low demand, the company would not be able to recover the cost of production and would have a loss of Rs. 5,00,000 annually.

(iii) If the present facilities are upgraded, then in case of high demand, there will be an annual profit of Rs. 15,00,000. The low profits are on account of penality imposed due to inability of the company to meet high demand. In case of moderate demand, the cost of lost sales will not be very high and the company would be able to make profit of Rs. 20,00,000 per annum. If the demand is low, then the demand will be in consonance with the production and the company would be making a profit of Rs, 30,00,000 annually.

The management is interested in knowing the best cousre of sction that it should take to maximize the profit.

Sol: We draw the decision tree of the problem

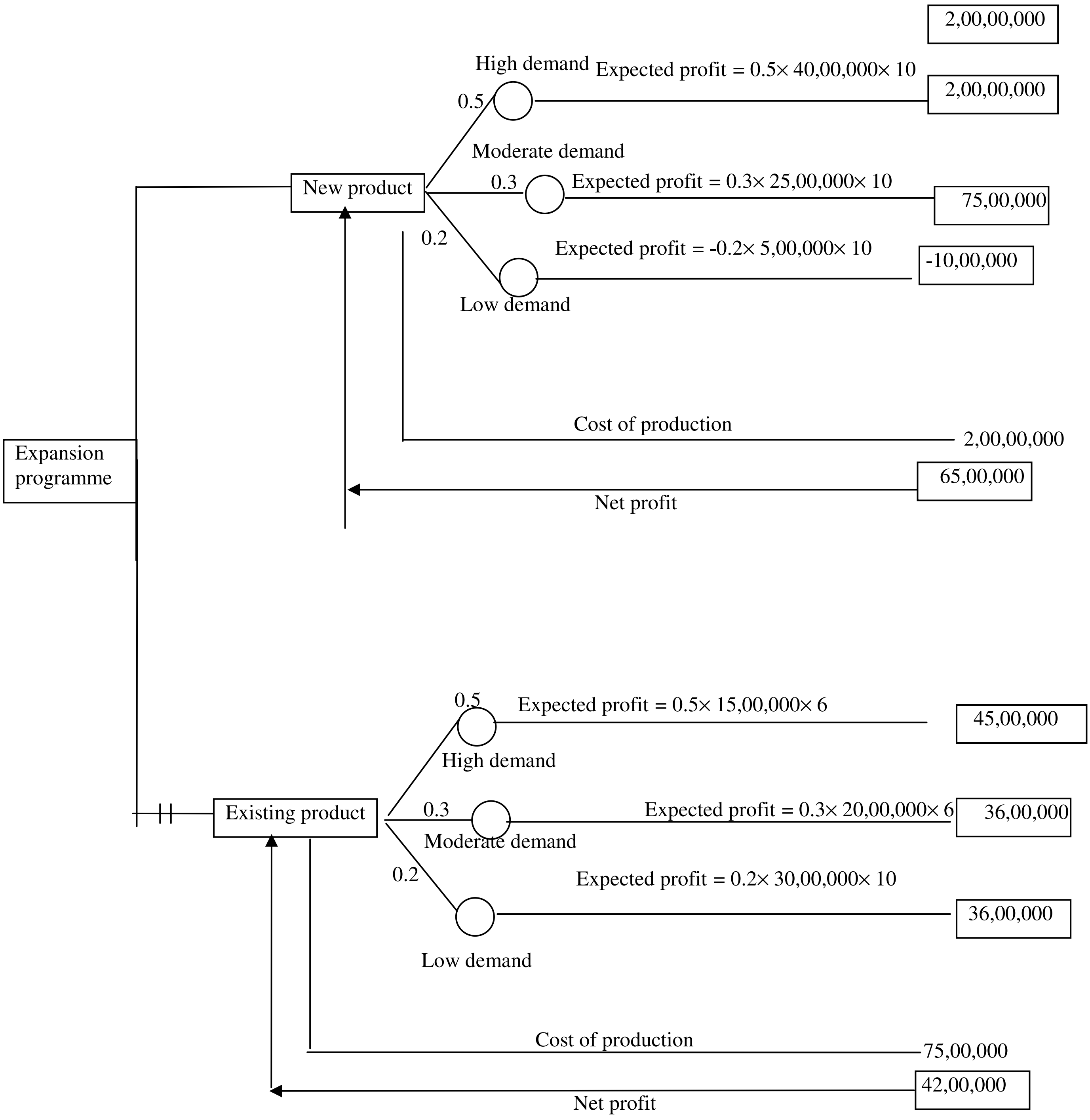

Fig. 5.12

Expected net profit of the first alternative is Rs. 65,00,000 and that of the second alternative is Rs. 42,00,000 for the second option. Hence the firm should go for thr intrduction of the new prodruct. (The branch corresponding to the second option has been prunned.)

Example2: Consider the case of a software firm, which has developed a new software which will assist mathematical researchers in developing the theoretical aspects of their research. Most of the softwares that have been developed till now are useful for analytical purposes but the proposed software will be helpful in designing the experiment and data generation also. The firm feels that there is a probability of 0.6 that the software will be successful in the market. However, it feels that there are 40% chances that the software will not be a success. If the software is successful, it will yield the firm a tremendous margin of Rs. 5,00,00,000 per annum for next 5 years. But, as the software development is a time-consuming and expensive task, in case of a failure, the firm will have cumulative losses of Rs. 2,00,00,000 in the next five years.

The firm has a safer option also. It can engage three experts who would use the software on a trial basis and give their opinion about the success of software in the market. This procedure will cost the firm Rs. 5,00,000. If the software will be a success in the market, then the probability that the experts will opine positively is 0.5. If the software fails in the market then the probability of positive opinion is 0.15. The firm wants to decide its future course of action.

Sol: The two alternatives before the firm are:

(i) Do not engage experts .

(ii) Engage experts and go by their opinion;

In this case, if experts are not engaged the expected pay-off depends upon the probabilities of success and failure. However, if the experts are engaged then their opinion is a function of the probabilities already estimated by the firm, i.e., this is a case of Bayesian estimation.

We now draw the decision tree of the problem

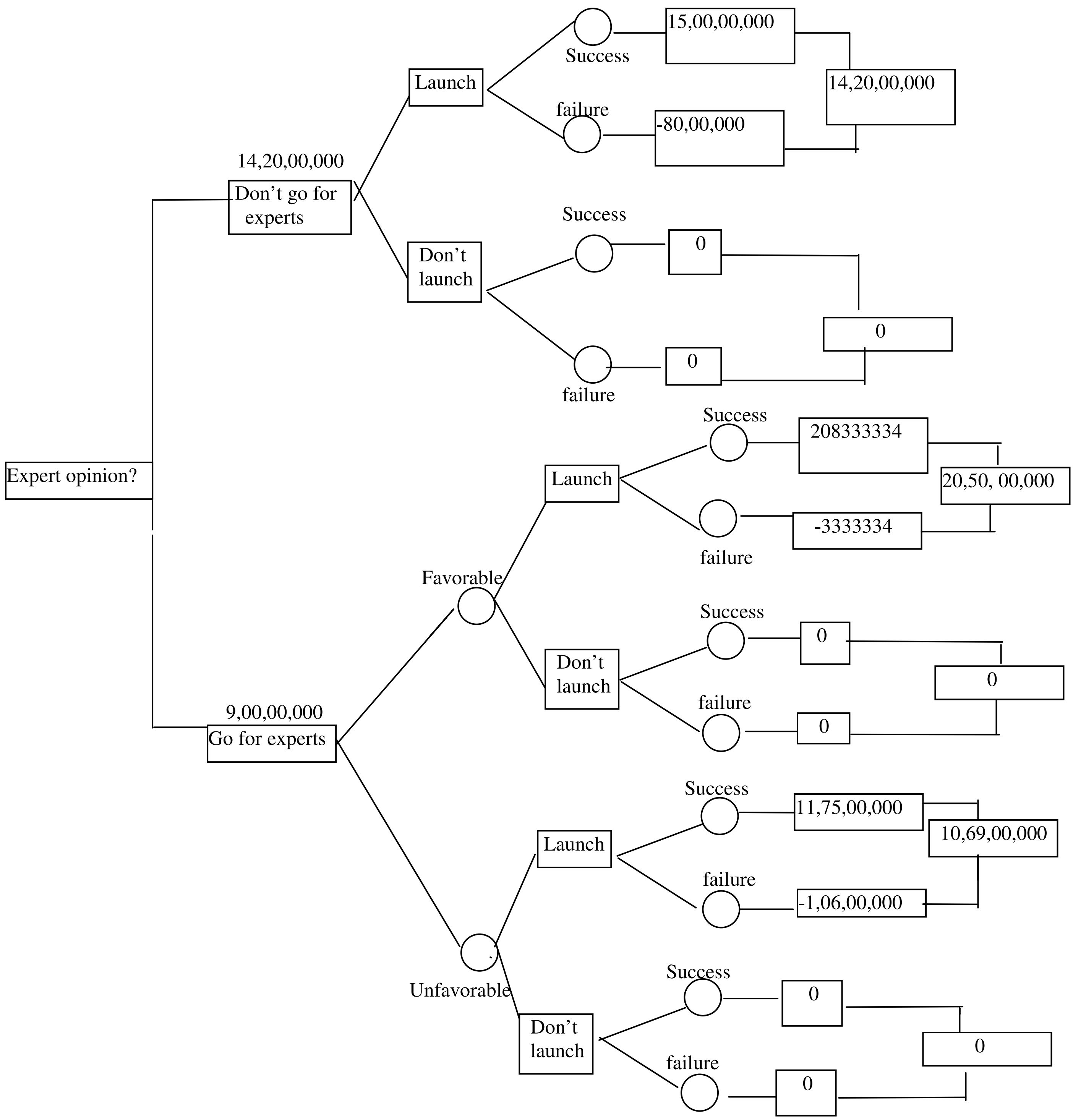

Fig. 5.13

We, now, examine and evaluate various alternatives

(i) Do not go for experts:

Table 5.4

Action	State of nature	Probability	Conditional pay-off (Rs.)	Expected value (Rs.)
Launch	Success	0.6	25,00,00,000	15,00,00,000
	Failure	0.4	-2,00,00,000	-80,00,000
				14,20,00,000
Do not launch	Success	0	0	0
	Failure	0	0	0
				0

(ii) Go for experts:

Table 5.5

State of nature	Action	State of nature	Probability	Expert's probability	Joint probability	Posterior probability
Favorable	Launch	Success	0.6	.5	.3	.3/. 36=5/6
		Failure	0.4	.15	.06	1/6
					.36	1
	Do not launch	Success	0	0	0	0
		Failure	0	0	0	0
					0	0
Unfavorable	Launch	Success	0.6	.5	.3	.3/. 64 =. 47
		Failure	0.4	.85	.34	.34/.64=. 53
					.64	1
	Do not launch	Success	0	0	0	0
		Failure	0	0	0	0
						0

Expected returns when experts are engaged

Table 5.6

State of nature	Action	State of nature	Posterior probability	Expected returns (Rs.)
Favorable	Launch	Success	5/6	5/6×(25,00,00,000) = 2,083,33,334
		Failure	1/6	1/6×(-2,00,00,000) -33,33,334
			1	**20,50, 00,000**
	Do not launch	Success	0	0
		Failure	0	0
			0	0
Unfavorable	Launch	Success	47	.47×(25,00,00,000) = 11,75,00,000
		Failure	53	-1,06,00,000
			1	**10,69,00,000**
	Do not launch	Success	0	0
		Failure	0	0
			0	0

Thus if the firm engages experts and they approve the software it is worth to launch it.

Finally, we calculate the expected value of engaging the experts:

Expected value of engaging the experts = 0.36(25,00,00,000)+. 64(0)

= Rs. 9,00,00,000

Expected value of not engaging the experts = Rs. 14,20,00,000

The optimal policy is that the firm need not go for experts and should launch the software commercially.

Example 3: A pharmaceutical company has developed a drug for controlling high blood pressure. On the basis of the past experience, the company knows that if the drug were successful, it would have an expected gross return in tunes of Rs.50, 00,000. But if the drug is unsuccessful, the expected gross returns will be Rs. 15, 00,000. Similar drugs launched in the past have experienced a success rate of 40%. The costs associated with drug have been estimated to be Rs. 20, 00,000.

The company wants to decide whether or not to launch the drug. Before, taking any decision, the company may test market the drug. It can introduce the drug on a limited basis in the market to obtain the feedback before commercially launching the drug. The costs associated with this exercise are Rs. 2, 00,000. If the test market results are successful, the success rate of the drug in the commercial market is revised to be 0.75. The company expects a 70% favorable test market. The firm's utility curve for money is given below.

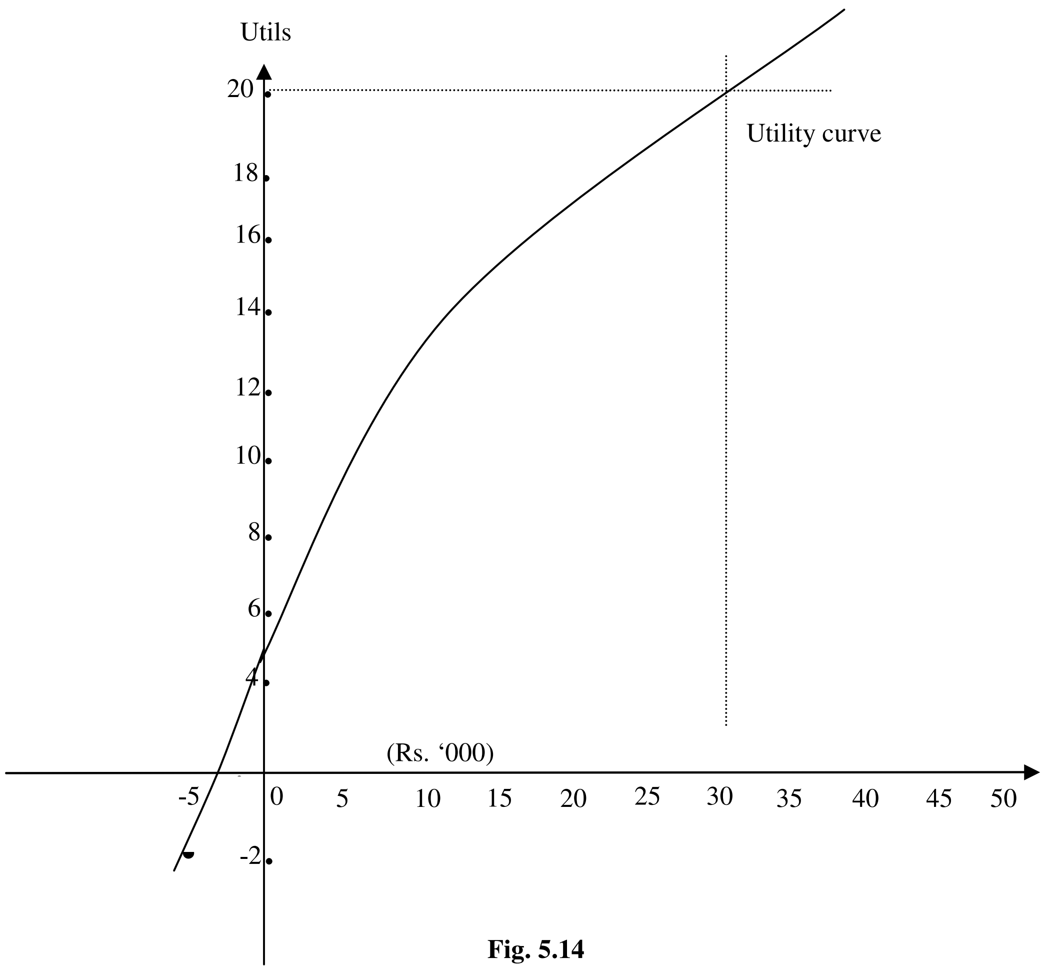

Fig. 5.14

Determine the optimal course of action.

Sol: The decision tree of the problem is drawn below:

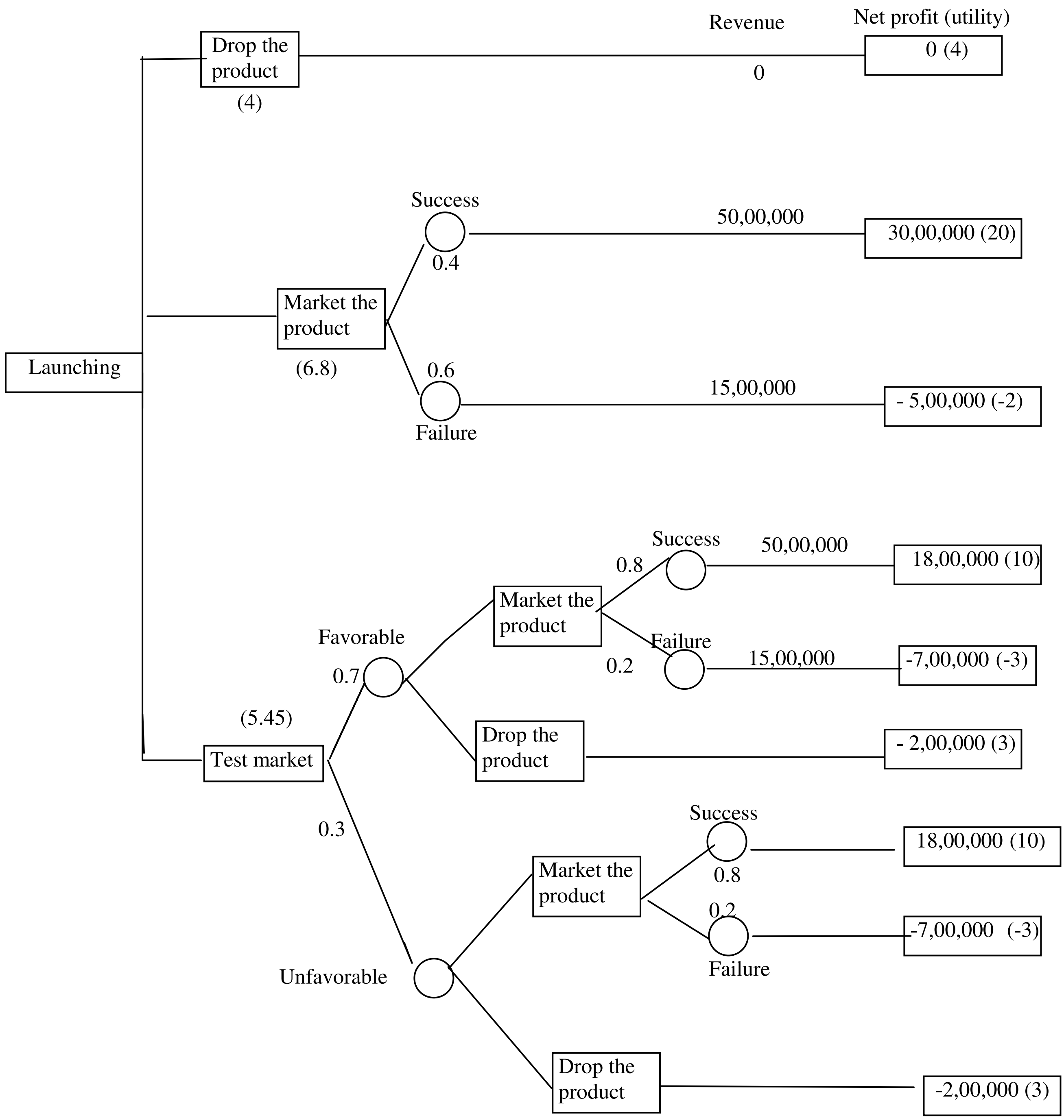

Fig. 5.15

The expected monetary values and the net profits are calculated as follows:

(i)

Table 5.7

Action	State of nature	Probability	Revenue (Rs.)	Net Profit (Rs.)	Net Profit Utility
Drop the product			0	0	5
Market the product	Success	0.4	50,00,000	30,00,000	20
	Failure	0.6	15,00,000	-5,00,000	-2
					Expected utility = .4(20)-.6(2) = **6.8**

Test market the product

Table 5.8

State of nature	Probability	Action	State of nature	Posterior probability	Revenue (Rs.)	Net Profit (Rs.)	Net Profit Utility
Favorable	0.7	Market	Success	.8	50,00,000	18,00,000	10
			Failure	.2	15,00,000	-7,00,000	-3
			Expected utility				**7.4**
		Drop			0	-2,00,000	-0.5
Unfavorable	0.3	Market	Success	.3	50,00,000	18,00,000	10
			Failure	.7	15,00,000	-7,00,000	-3
			Expected utility				**0.9**
		Drop			0	-2,00,000	-0.5
	Expected utility						.7(7.4)+. 3 (0.9) = **5.45**

Since expected utility of launching the product directly is more then the expected utility of test marketing the product the firm should go directly for marketing the product.

5.5 Sensitivity analysis

Sensitivity analysis refers to the studying the changes in the output variables of the model as a result of slight variations in the input parameters of the model.

Sometimes it is possible to formulate the model but some of the parameters of the model may not be known with certainty, either due to shortage or lack of appropriate data. In such situations, if some information about the outcomes in the past is known, the same can be used to try various possible combinations of the parameters of the model so that the possible input combination can be obtained.

If it seems that the model outputs are very sensitive to (a) particular variable(s), it may be worthwhile to obtain a range of such parameters in place of a point estimate so that the model doesn't fail in case of slight variations in the parameters. In such situations also, sensitivity analysis is done.

In sensitivity analysis, in general one parameter is varied while keeping others at a constant level and then the variations in the output are obtained. These variations can be drawn on graph to study the direction and the magnitude of the change.

Consider the software company problem

Suppose that p is the probability that the software will be successful. Then, if the company doesn't go for experts, then the expected pay-off of the company is

$$\begin{aligned} E(\text{ pay-off }) &= p(25,00,00,000) - (1-p)(2,00,00,000) \\ &= 27,00,00,000p - 2,00,00,000 \end{aligned}$$

As long as this expected pay-off is positive, it is worth to launch the software, i.e.,

$$27,00,00,000p - 2,00,00,000 \;>\; 0$$

$$\Rightarrow \qquad p \;>\; \frac{2}{27} = 0.07$$

For different values of p, the expected pay-off has been shown in the following graph:

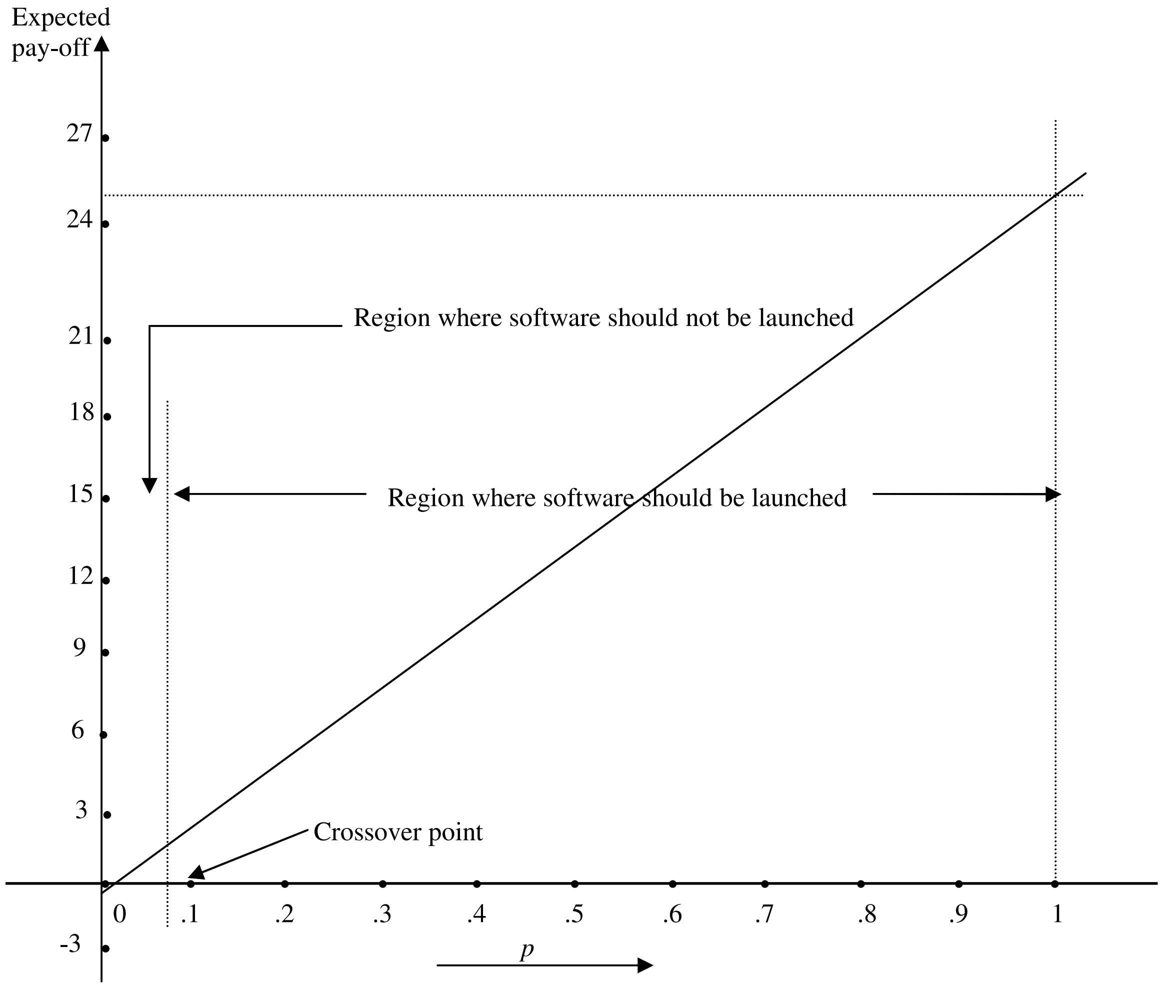

Fig.5.16

Crossover point is that point where the direction of the decision changes.

The graph shows that the decision is very sensitive to p, since $p = 0$ is giving a huge loss but $p = 1$ is giving a tremendous profit. Still the region where software can be launched is quite large.

5.6 An integrated example

In India, farming mainly depends on Monsoons. If Monsoons are on time and adequate in quantity, a good harvest can be expected. (Although harvesting depends upon other factors also). However, rain gods are not always so merciful. An alternative arrangement of the water is under-ground water. But the situation is that under-ground water resources are also receding. A farmer Mohan, as many others, is facing this problem.

He has a piece of land where he wants to dig a tube well, which would help in irrigating his fields in case of insufficient rains. However, under-ground water in this part of India is not very plentiful and he has estimated that there are 25% chances of finding the water. The cost of digging the land is very high, i.e. Rs. 1, 00,000. This high cost is on account of the fact that water, if any, will be several hundreds feet below the level and digging may have to be done at several points. If water is found, the resulting expected profit will be Rs. 7, 00,000 annually. However, in case of failure, the whole exercise will be futile.

Another rich farmer Gopal in the vicinity wants to acquire that land for Rs. 90,000 so that Mohan is assured a certain pay-off of Rs. 90,000.

Now, Mohan has to decide his course of action.

Mohan has an intelligent daughter Hira, who is interested in decision theory and knows it is possible to evaluate different options objectively. She offers help to her father. Mohan asks her to carry a detailed analysis of the problem. We present below the analysis done by Hira.

(i) Stage I:

$$\begin{aligned} \text{E(digging)} &= 0.25(7,00,000) + 0.75(-1,00,000) \\ &= 1,75,000 - 75,000 \\ &= \text{Rs.}1,00,000 \end{aligned}$$

$$\begin{aligned} \text{E(selling)} &= 0.25(90,000) + 0.75(90,000) \\ &= \text{Rs.}90,000 \end{aligned}$$

Obviously, the expected pay-off of digging is more than the expected pay-off of selling.

(ii) Stage II:

But Hira is well aware of the fact that a statement like "25% chances of finding the water" is very rigid. A more flexible statement would be to predict a range for probability. So she decides to carry out a sensitivity analysis for the probability of finding water.

If p is the probability of finding water, then the expected pay-off from digging is given as

$$\begin{aligned} \text{E(digging)} &= p(7,00,000) + (1-p)(-1,00,000) \\ &= 8,00,000p - 1,00,000 \end{aligned}$$

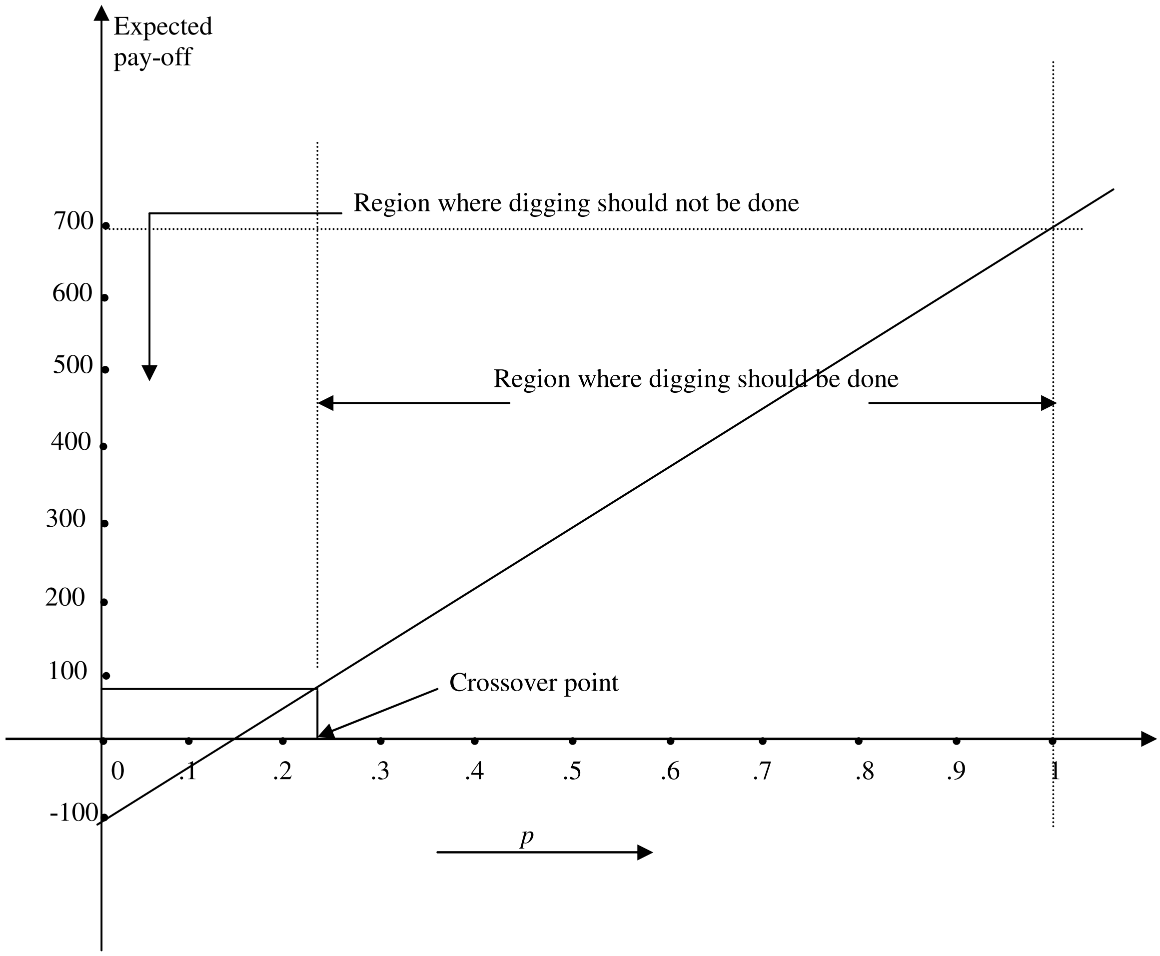

Fig. 5.17

The crossover point is where the expected pay-off from digging is same as the expected pay-off from selling, i.e.

$$8,00,000p - 1,00,000 = 90,000$$

$$\Rightarrow p = \frac{1,90,000}{8,00,000} = 0.2375$$

Thus Hira's advice is that her father should sell the land if the probability of finding water is less than 0.2375 and should go for finding water if this probability is more than 0.2375.

(iii) Stage III:

Also, Hira has come to know that by spending some more money, she can get some more information about the nature of her field. If she gets the soil of her field tested, and some more experiments are conducted in the field then the improved probability estimates will be according to the following table:

Table 5.9

Results	State of nature	
	Water	No water
Favorable	0.6	0.2
Unfavorable	0.4	0.8

Now, she has to decide whether or not to go for testing the soil. In the light of new information, she revises her probability estimates:

Table 5.10

State of nature	Prior probability	Result of testing	Conditional probability	Joint probability	Posterior probability
Water	0.25	Favorable	0.6	0.15	$\frac{.15}{.15+.15} = \frac{1}{2}$
		Unfavorable	0.4	0.10	$\frac{.10}{.10+.60} = \frac{1}{7}$
No water	0.75	Favorable	0.2	0.15	$\frac{.15}{.15+.15} = \frac{1}{2}$
		Unfavorable	0.8	0.60	$\frac{.60}{.10+.60} = \frac{6}{7}$

The expected pay-offs from the decision of getting the soil tested are calculated as follows:

Table 5.11

State of nature	Decision	Expected pay-off (Rs.)
Favorable	Dig	$\frac{1}{2}(7,00,000)-\frac{1}{2}(1,00,000)-30,000 \;=\; 2,70,000$
	Sell	$90,000-30,000 \;=\; 60,000$
Unfavorable	Dig	$\frac{1}{7}(7,00,000)-\frac{6}{7}(1,00,000)-30,000 \;=\;$ -15,714
	Sell	$90,000-30,000 \;=\; 60,000$

According to Hira, the optimal policy should be:

(i) Retain the land if the results are favorable; and

(ii) Sell the land if the results are unfavorable.

(iii) **Stage IV:**

When she discussed these findings with her father, Mohan asked her to find out whether it was worth to test the soil or not. Technically this amounts to saying whether the expected value of sample information is more than the cost of obtaining it or not.

Hira calculates the expected value of sample information and expected gain from sampling:

EVSI = (expected pay-off | favorable outcome) P(favorable outcome)

+ (expected pay-off | unfavorable outcome) P(unfavorable outcome)

= .3(3,00,000)+.7(90,000)

= 1,53,000

EV without sampling = 1,00,000

Expected gain from sampling = Rs. 53,000

The cost of testing = Rs. 30,000

$\Rightarrow$ Expected gain from sampling > The cost of testing

Hence Hira decides to go for testing the soil.

She is also interested in knowing what she is loosing due to not having the perfect information. For that, she calculates the expected value of the perfect information as follows:

Table 5.12

Action	State of nature	
	Water	No water
Digging	700	-100
Selling	90	90
Probability	0.25	0.75
Expected pay-off = 0.25(7,00,000)+0.75(90,000) = 2,42,500		

EVPI = Expected pay-off of perfect information - Expected pay-off without perfect information

= 2,42,500 - 1,00,000

= Rs. 1,42,500

Thus, as long as the cost of getting information does not exceed Rs. 1,42,5000, she can keep on spending to get more information.

(v) Stage V: Optimal decision policy

Hira now calculates the expected pay-off of testing as follows:

Expected pay-off of testing = (expected pay-off | favorable outcome) P(favorable outcome)

+ (expected pay-off | unfavorable outcome) P(unfavorable outcome)

= .3(2,70,000)+.7(60,000)

= 1,23,000

EV without sampling = 1,00,000

$\Rightarrow$ Expected gain from testing > Expected gain without testing

The policy, then should be

(i) Do the testing.

(ii) If the results are favorable, dig the land

(iii) If the results are unfavorable, sell the land.

Using this policy, the expected pay-off would be Rs. 1, 23,000 for the first year.

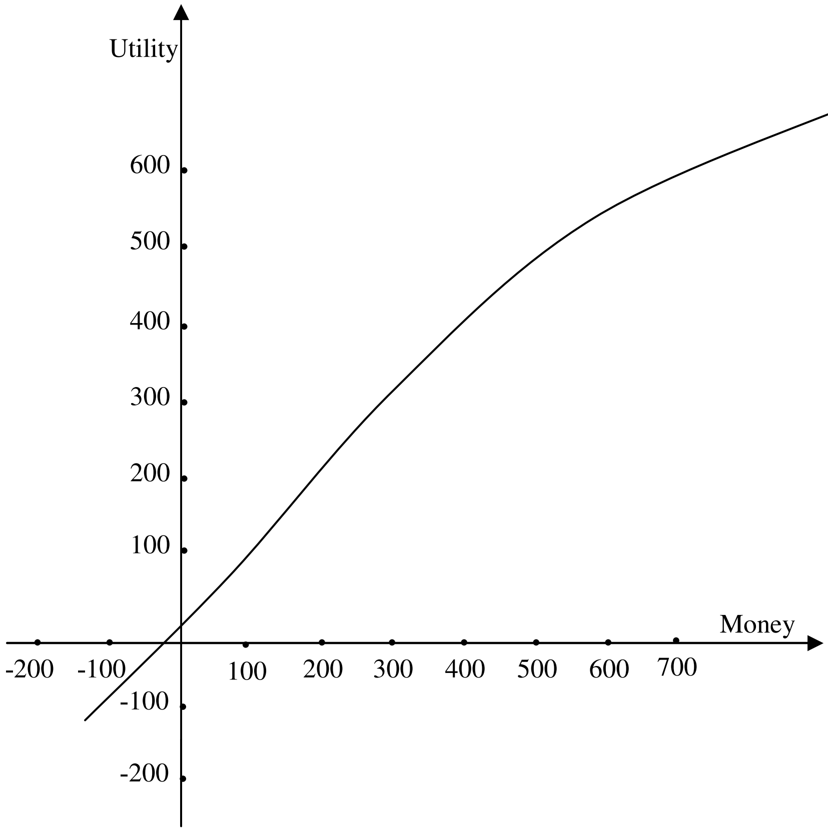

Fig. 5.18

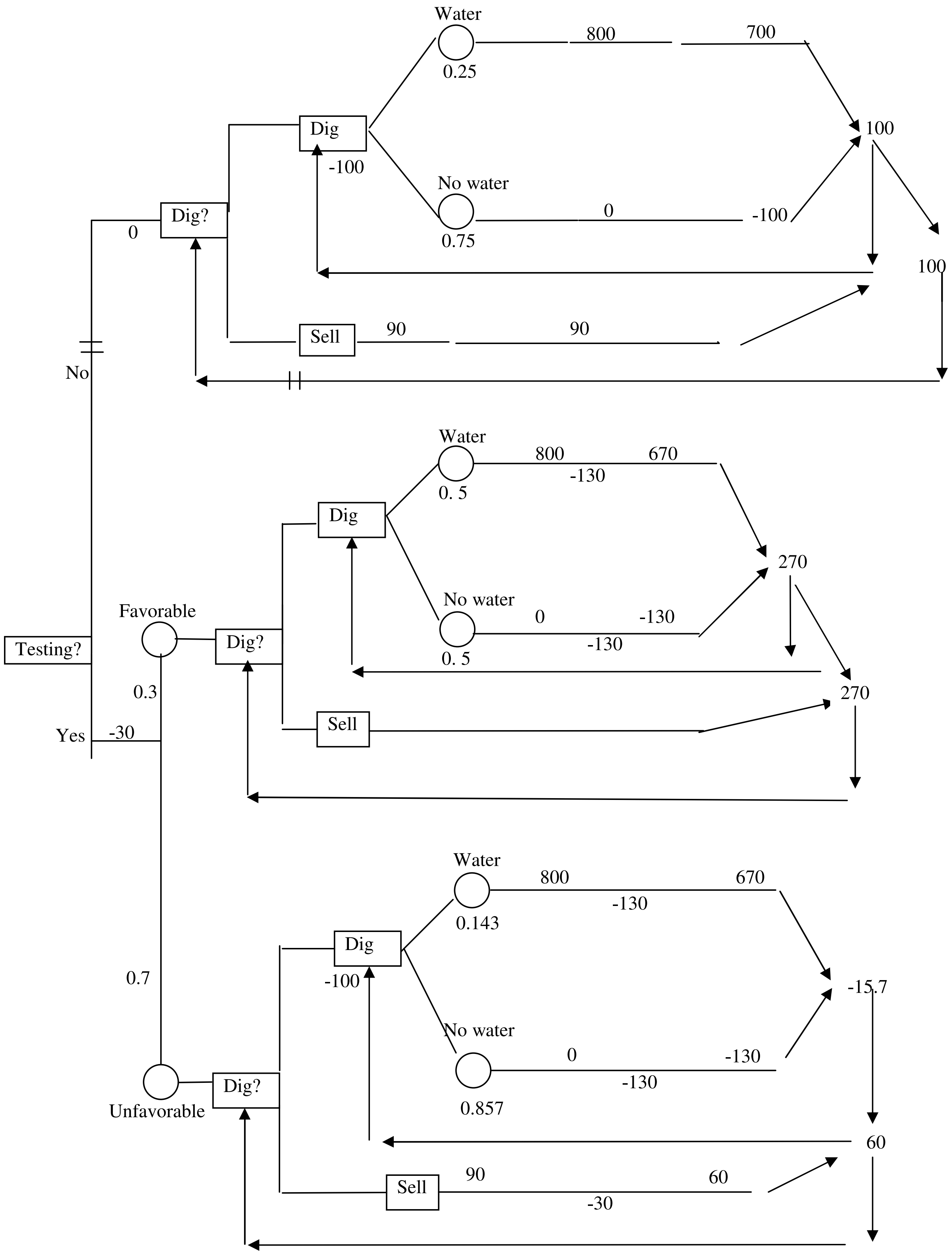

Fig. 5.19 Decision tree using utilities when the utility curve of Mohan is given above.

Problems

1. In the oil extraction problem, consider the following data:

Cost of testing	Rs. 2, 00,000
Cost of drilling	Rs. 10, 00,000
Profit per barrel	Rs. 12

Find the optimal course of action of the firm.

2. SIS Technology is a company operating cyber cafés in a city. For an hourly fee of Rs. 12.00, the company provides access to a personal computer and Internet facility. The hourly variable cost to the company has been estimated to be Rs. 2.50. Now the company is planning to start a new café. The demand schedule for the computers (per hour) has been estimated as follows:

Table 5.13

Number of computers	15	16	17	18	19	20
Probability	0.10	0.15	0.25	0.30	0.15	0.15

In order to maximize its profit, how many PCs should be installed by the company? Find expected value of perfect information.

3. A BPO center in the city hires executives at an hourly rate of Rs. 175. The management of the center has estimated that the annual requirement of the executive hours is as follows

Table 5.14

Number of hours	10,000	12,000	15,000	18,000	20,000
Probability	0.20	0.25	0.30	0.15	0.05

If the revenue generated per executive hour is Rs. 210, find

(a) The number of executives that the center should hire.

(b) The expected value of perfect information.

It is known that the executives work 45 hours a week with a two-week annual vacation.

4. Contemporary Periodicals is a bookstore selling quarterly journals on current affairs. These journals are highly demanded by students preparing for various competitive examinations. A new journal costs Rs. 18 to the store and it fetches Rs. 26 to the store. In the second month of its publication, the journal would fetch only Rs. 20. However when a new addition comes, the left over stock can only be sold for Rs. 8 per journal. The owner of the store has estimated the following demand schedule for a new addition:

Table 5.15

Number of copies required	1000	1100	1200	1300	1400	1500
Probability	0.18	0.19	0.21	0.15	0.14	0.13

The order for new addition must be placed 20 days prior to its publication. Find the optimal number of copies to be ordered so as to maximize the profit of the store.

5. New India Times is a popular newspaper in a city. A newsstand sells this newspaper according to a normal distribution with mean 200 and standard deviation 50. The selling price of a copy is Rs. 2 and it costs Rs. 1.50 to the newsstand. Unsold copies can be sold for 20 paise per copy. In order to maximize its profit, how many copies should be ordered by the newsstand?

6. A firm has several investment proposals before it. The target rate of return of the firm is 10%, above which its utility rises very fast. Between a rate of 0% and 10%, the rise in utility is just marginal above 0, and below 0%, it declines very rapidly. If the amount that the firm wants to invest is Rs. 25,00,000, draw the utility curve of the firm.

7. Consider the following information

   (a) An indifference between a sure sum of Rs. 20,000 or a 90:10 bet between a gain of Rs. 30,000 and a loss of Rs. 30,000.

(b) An indifference between a sure sum of Rs. 10,000 or an 80:20 bet between a gain of Rs. 20,000 or nothing.

(c) An indifference between a sure loss of Rs. 10,000 or a 40:60 bet between a loss of Rs. 20,000 or nothing

If the sum Rs. 30,000 has utility 100 and –Rs. 20,000 utility 0, draw the utility curve. What can you say about the nature of the investor?

8. A mutual fund manager is considering the following investment options for a part of funds available with him

(a) To invest in highly volatile entertainment industry. In this investment with probability 0.5, he may loose his money, with probability 0.3, the profit will be of tunes of 30%, and with probability 0.2 the profit will be 75% of the investment.

(b) To invest in real estate. If invested in this area, he will get returns according to the following schedule:

Table 5.16

Rate of return (%)	10	20	50	70	80
Probability	0.10	0.25	0.30	0.20	0.15

(c) To fix deposit in a bank at a sure return of 6%.

(i) Construct a decision tree to help manager decide his course of action.

(ii) What should be return on fixed deposit before he would opt for it?

(iii) What are the values of perfect information for first two options?

9. A fashion house is planning to introduce a new fabric in the market. It has two options before it. The first option is to start full-fledged production with the new stuff and the second option is to introduce the stuff at a limited scale. If the results of limited production are promising, the full-fledged production may be undertaken. If the limited production does not show very encouraging results, it can still be continued. The expected annual profits are as follows

Table 5.17

Production	Market acceptance	Annual profit (Rs., 00,000)
Full scale	High	50
	Low	-6
Limited	High	7
	Low	2

There are 40% chances of market acceptance to be high if the full-scale production is undertaken. If limited production is undertaken, the chances of consumer acceptance are 35%. However if limited production is successful, full-scale production will be successful with probability 0.90. If the limited production results in a low market acceptance, the full-scale production will be successful with probability 0.20. What should be the courts of action of the fashion house?

10. A pharmaceutical company is planning to introduce a new drug for cure of Tuberculosis. The following estimates have been made in this regard:

Table 5.18

Level of success	Probability	Annual Profit (Rs. 00'000)	
		Limited production	Full market production
Low	0.3	30	-5
Average	0.5	10	10
High	0.2	15	80

(a) Analyze the data to help company to reach at a decision.

(b) Before launching the production at a later stage on a full scale, an option with the company is to seek experts' opinion. The cost of experts' opinion is Rs. 1,00,000. The opinion says that the chances of success are 30% if limited production results in a low success; 50% if limited production results in a moderate success; and 90% if limited production results in a high success. In light of this information, what should be the decision of the company?

11. A Governmental funding agency is to sponsor NGOs working in the filed of rural employment. The maximum amount of sponsorship that can be offered is Rs. 15,00,000. The selection process of the NGOs which has been used till now, has classified the NGOs according to their performance as follows:

Table 5.19

Class	Proportion	Income generated (Rs.)
Poor	25%	-5,00,000
Average	50%	10,00,000
Good	20%	20,00,000
Excellent	5%	50,00,000

Now the sponsoring agency is planning to take help of a professional group, which would rate agencies (independent of ratings of the governmental agency) according to their efficiency. Three level of efficiency are C, B and A in increasing order. The following results have been obtained while relating the two classifications

Table 5.20

Classification of the professional group	Classification of the governmental agency			
	Poor	Average	Good	Excellent
A	0.10	0.10	0.40	0.60
B	0.20	0.80	0.40	0.30
C	0.70	0.10	0.20	0.10

(a) Using Bayes' theorem, determine whether or not, should the professional group be engaged?

(b) Does the hiring of the professional group really affect the true category of NGO?

(c) What is the maximum amount that can be paid to the professional group?

(d) If the professional group is to be paid Rs. 50,000, what should be the decision.