Chapter 4

Decision Making Using Utility

Key words: Utility theory, utility curve, St. Petersburg's paradox, risk aversion, risk neutrality, risk seeking.

Suggested readings:

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2. Gupta P.K. and Mohan M. (1987), **Operations Research and Statistical Analysis**, Sultan Chand and Sons, Delhi.
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4.1 Introduction

The basis of decision-making under uncertainty or under risk is the maximization of the expected pay-offs (in monetary terms) or minimization of the expected loss. Consequently, a strategy, which returns the maximum benefit, must be selected. But this approach may not be advisable always and at times decisions may have to be taken which are returning lesser benefits. Such a selection may be justified on the basis of the rationality of behavior. For example, consider the case of a medical insurance. Here a person gets insured for a disease e.g. a cardiac problem that he may never acquire in his life. Still, looking at the prevalence of the disease, such insurance is advisable.

Similarly a vehicle may never be indulged in an on-road accident; still it is compulsory to get it insured. Compensation to some extent in such situations is in terms of 'no-claim' bonus given to the owner of the vehicle.

In such situations, the decision-maker is choosing an alternative, which is monetarily expensive but is being chosen for its usefulness. This is the concept of utility theory. Utility theory helps us to take decisions not from the monetary point of view, but from the rationality of behavior.

Here is a famous problem which is known as **St. Petersburg's paradox**.

Consider the game of tossing a fair coin which is tossed until the first head appears. The game continues and for each trial, the player is paid an award of \$2 at the end of the game. This means if the game ends at the first trial itself, the player would be paid \$2 (2^1), if the game ends at the second trial, he would get \$4(2^2), at third trial \$8(2^3) and so on. Since the coin is a fair coin, the probability of turning up a head is same as that of a tail, i.e., ½. What is the expected value of the game for the player? We calculate the expected value of the game as follows:

$$\text{Expected value of the game} = \sum_{n=1}^{\infty} 2^n \left(\frac{1}{2}\right)^n$$

$$= \sum_{n=1}^{\infty} 1^n = \infty\textbf{!!!}$$

The player can expect an infinite amount to earn for the game so he should be willing to put any amount at stake in the game as he can surely expect the return of his money. But would he do so! Surely not. In fact, in reality, no rational player would be interested to put at stake even \$20. What is the source of this conflict?

In this case, the concept of expected value of game is flawed. The games are not played according to the expected pay-off but according to their expected utility. If expected value of money (EMV) is infinite, it does not mean that utility of the game is also infinite.

Utility theory proposes that an individual takes decisions so as to maximize his personal utility i.e., the satisfaction that he derives from the fulfillment of his own personal wants. For example, in case of a medical insurance, the assurance that the person will get help in the time of need may be more fulfilling than sparing some money annually to pay the premium although that emergency time may never come.

Utility also depends upon the money at stake. Consider the case of a lottery game. Various amounts are put on stake for different bets. Even if the chances of winning are the same, say, Rs. 50,00,000 at a bet of Rs. 10,000 and Rs. 10,00,000 for a bet of Rs. 2,000, most of the people would opt for the second gamble, as in case of loss, they will be loosing a comparatively smaller amount.

Utility is a function of desirability. Hence, utility of the same object may be different for different individuals. When you discarded-off your old motorbike it had lost its utility for you, but it has high utility for the person who purchased it. Similarly a person may put high value on electronic gadgets, some other may have an inclination towards traveling, and a third category may put high value for antiques or jewellery. Thus utility is a subjective criterion.

In spite of subjective nature of utility, it is possible to treat it mathematically. We can define a **utility function** as a functional relationship between utility and its monetary value. When this relationship is represented graphically, we get what we call as a **utility curve**.

Mathematical measurement of utility makes a sense in the situation when we are measuring relative utilities. Absolute utilities make no sense. What value would you assign to a rail-travel to a place that is not connected to your place by any other mean of transport? You can assign it any value you wish without making any difference. But if the place is connected by road also, then the utility of a rail travel may be more or less than the utility of traveling by road.

Utility may be measured by considering the monetary pay-off associated with a decision. Mathematically utility resides on some basic principles or axioms called the **axioms of coherence**.

Axioms of coherence Axioms of coherence rationalize the behavior of the decision maker in the sense that if his utilities satisfy these axioms then he would make decisions so as to maximize his expected utility. We define these axioms as follows:

(i) **Utilities are transitive**: If corresponding to three decisions d_1, d_2, d_3 three are three pay-offs R_1, R_2, and R_3 respectively such that $R_1 > R_2$ and $R_2 > R_3$, then $R_1 > R_3$.

(ii) **Utilities can be substituted**: If a person is indifferent to two pay-offs R_1 and R_2, then it will make no difference if he is substituting the utility R_1 of to utility of R_2 or vice versa.

(iii) **Preferences can be ordered**: For any two given pay-offs R_1 and R_2, the user is able to decide whether R_1 is to be preferred to R_2 or vice versa or if the user is indifferent to both of them.

(iv) **Inadmissible actions (decisions) can be eliminated**: If for two pay-offs R_1 and R_2, R_1 is preferred to R_2 and R_3 is some other pay-off then in the following two bets, bet 1 will be preferred to bet 2:

Bet 1: Receive R_1 with probability p
Receive R_3 with probability 1-p

Bet 2: Receive R_2 with probability p
Receive R_3 with probability 1-p

This is because receiving R_2 in bet 2 is an inadmissible action as $R_1 > R_2$.

There re two basic principles of utility:

(i) A pay-off R_1 is preferred to a pay-off R_2 iff

$$U(R_1) \quad > \quad U(R_2)$$

and A pay-off $R_2$is preferred to a pay-off R_1 iff

$$U(R_1) \quad < \quad U(R_2)$$

If $U(R_1) \quad = \quad U(R_2)$ then neither of them is preferred where $U(R_i) \ \ (i=1,2)$ is the utility of R_i.

(ii) In case a user is indifferent between a sure pay-off R_1 and a pay-off R_2 with probability p and a pay-off R_3 with probability 1-p, then utility of R_1 is a linear function of utilities of R_2 and R_3, i.e.

$$U(R_1) \quad = \quad pU(R_2)+(1-p)U(R_3)$$

If any utility function U satisfies these principles then any linear function $\alpha+\beta U(R_1)$; $\beta>0$; α and β constants, of U will also satisfy them.

4.2 Construction of a utility curve

The basic assumption in the construction of a utility curve is that the decision maker is indifferent between two alternatives if they have the same expected utility. A utility curve may be constructed by plotting the value of utility against its monetary pay-off. Suppose that a person is indifferent to receiving a sure sum of Rs. 50,000 or a 50-50 bet of receiving Rs. 2,00,000 or nothing. If the certainty amount were more than Rs. 50,000, he would not choose to gamble. But, if the certainty amount were less than Rs. 50,000, he would prefer the gambling. Since the utility is meaningful when talked in relative terms, we start the process by assigning arbitrary values to the utilities of the given amounts. Then we can calculate the utility values of the other monetary amounts in relation to these assigned utilities. Suppose that we assign a value of 0 utils (the unit of utility) to the sum of Rs. 0 and 100 utils to the amount of Rs. 2,00,000. Then since the person is indifferent to a 50-50 bet of receiving Rs. 2,00,000 or nothing; or Rs. 50,000, so the relative utility of Rs. 50,000 would be

$$0.5(0) + 0.5(100) = 50$$

Again, if he is indifferent between a 50-50 bet of receiving Rs. 2,00,000 and Rs. 50,000 and a sure sum of Rs. 1,00,000, then the utility of Rs. 1,00,000 is given by

$$0.5(50) + 0.5(100) = 75$$

Calculating the utility values for some more amounts and plotting theses points on a set of axes, we get the utility curve of this individual.

The utility curve can be extended to plot negative utilities also.

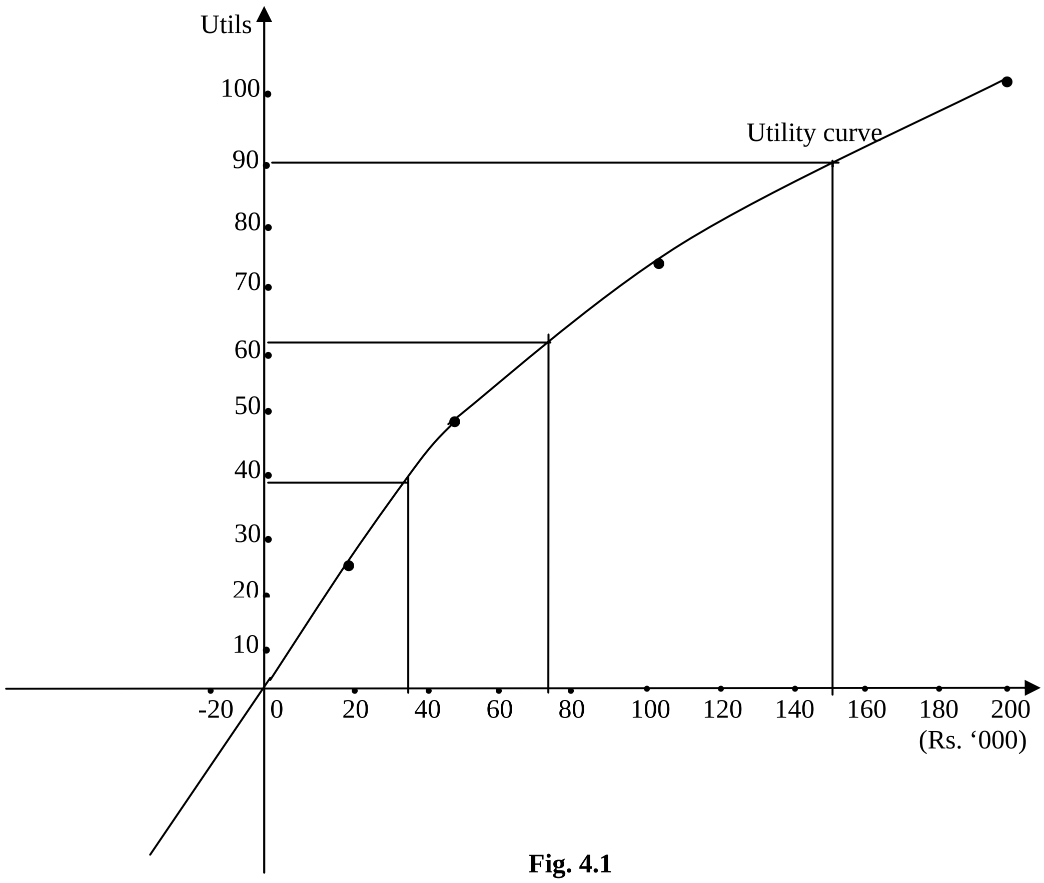

Fig. 4.1

Interpreting the utility curve

A utility curve is representation of one's behaviour towards risk aversion or risk adventure. Thus, the utility curves can very well be used to predict the choices of the individuals when they are presented with a set of risky alternatives.

The utility and the expected pay-offs are related but not necessarily linearly. We have seen examples of the circumstances (for example medical insurance) when expected pay-offs are zero or negative still utility favors the particular decision (purchasing of policy). Also we have seen that the value of utility changes with monetary value of pay-offs, i.e. for smaller pay-offs a decision may have higher utility than that for higher pay-offs (gambling games).
In general, utility of pay-offs zero is taken as zero (note that this is not a rule and the arrangement is just for convenience similar to the one when we assign value 100 to the highest utility. You can take any other utility to be 0 and derive other utilities accordingly.)

Consider the utility curve 4.1. We notice that the curve is concave downwards. In order to analyze the preferences of the individual, suppose that he is supposed to make a choice between the following two alternatives:

(i) A return of Rs. 35,000 with probability 0.6 or a return of Rs. 75,000 with probability 0.4.
(ii) A 35-65 bet of Rs. 1, 50,000 or nothing.

From the curve, we calculate the expected utilities of the two offers as follows:

(i) Expected utility of the first offer $= 0.6(39) + 0.4(60.75) = 47.7$
(ii) Expected utility of the second offer $= 0.65(0) + 0.35(90) = 31.5$

Since the expected utility of the first alternative is higher than the expected utility of the second offer, the individual will go for the first alternative.

Now, we calculate the expected pay-offs of the two alternatives:

(i) Expected pay-off of the first offer $= 0.6(35,000) + 0.4(75,000) = 51,000$
(ii) Expected pay-off of the second offer $= 0.65(0) + 0.35(1,50,000) = 52,500$

Thus, we notice that in spite of higher expected pay-off of the second alternative, the individual is still willing to go for the first alternative, because he is feeling safer when going for this alternative.

Thus the shape of the utility curve is related to the risk preferences of the decision maker.

Consider the following betting game:

A fair coin is being tossed. If a "head" turns up, the (active) player will receive Rs. 50 form the other player and if a "tail" turns up, he would pay Rs. 50 to the other player. If he quits the game, then he would not gain or loose anything.

Now, the expected pay-off (gain) of the active player is

$$EG = \frac{1}{2}(50)+\frac{1}{2}(-50) = 0.$$

But his expected utility (of playing the game) is

$$EU(\text{game}) = \frac{1}{2}U(G)+\frac{1}{2}U(L)$$

where

$U(G) = U(50) - U(0)$ is his net gain in utility if he gets a "head";

and

$U(L) = U(0) - U(-50)$ is his net loss in utility if he gets a "tail".

$U(0)$ is his utility if he does not play the game.

$$\Rightarrow EU(\text{game}) = \frac{1}{2}U(50)-\frac{1}{2}U(-50).$$

Also, the expected utility of not playing the game is

$$EU(\text{no game}) = U(0).$$

A player will play the game iff

$$EU(\text{game}) > EU(\text{no game})$$

$$\Leftrightarrow \frac{1}{2}U(50)+\frac{1}{2}U(-50) > U(0)$$

$$\Leftrightarrow \frac{1}{2}\left(U(50)-U(0)\right) > \frac{1}{2}\left(U(0)-U(-50)\right)$$

$$\Leftrightarrow U(G)- U(L) > 0$$

Thus the decision doctrine becomes:

(i) Play the game if $U(G) - U(L) > 0$.

(ii) Do not play the game if $U(G) - U(L) < 0$.

Three cases arise:

Case 1: Consider the following curve

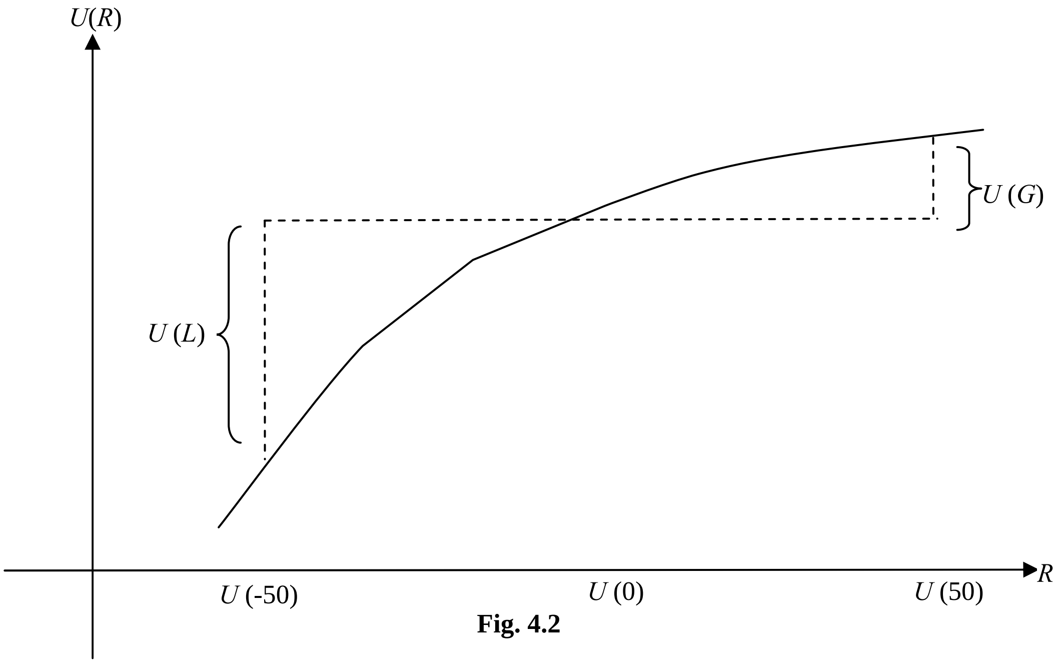

Fig. 4.2

In this case, $U(G)$ is less than $U(L)$ so that $U(G) - U(L)$ is negative. Such a player will not take the bet which corresponds to an expected monetary pay-off of value 0 and is called a **risk -averse**. The curve is that of a risk-averse player.

Case 2: Consider the following curve

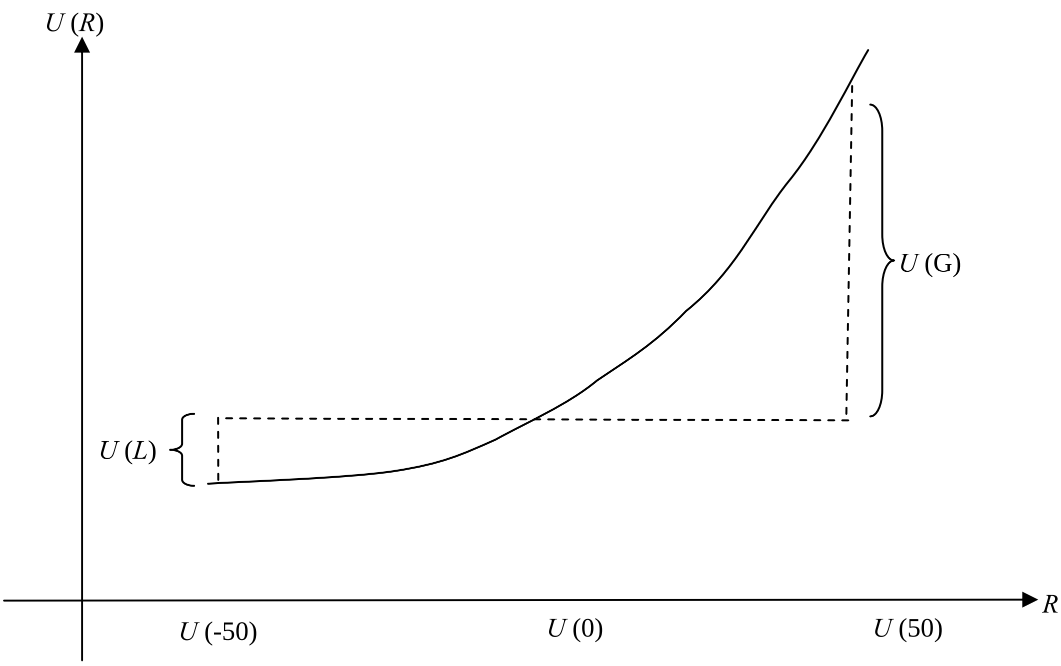

Fig. 4.3

In this case, U (G) is more than U (L) so that U (G) - U (L) is positive. Such a player will take the bet which corresponds to an expected monetary pay-off of value 0 or even negative monetary pay-off and is called a **risk -seeker**. The curve is that of a risk-seeking player.

Case 3: Consider the following curve

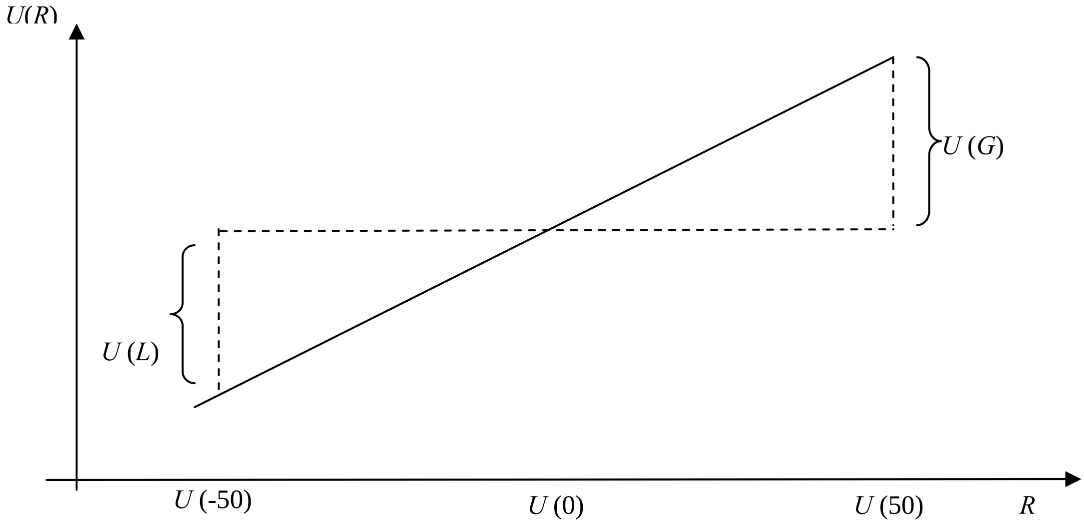

Fig. 4.4

In this case, U (G) is equal to U (L) so that U (G) - U (L) is zero. Such a player is indifferent to a bet which corresponds to an expected monetary pay-off of value 0 and is called a **risk -neutral**. The curve is that of a risk-neutral player.

Another form of a utility curve is exponential utility given as

$$U(R) \quad = \quad \alpha - e^{-\beta R}; \ \alpha \text{ and } \beta \text{ are constants.}$$

The graph of this type of utility function is as follows:

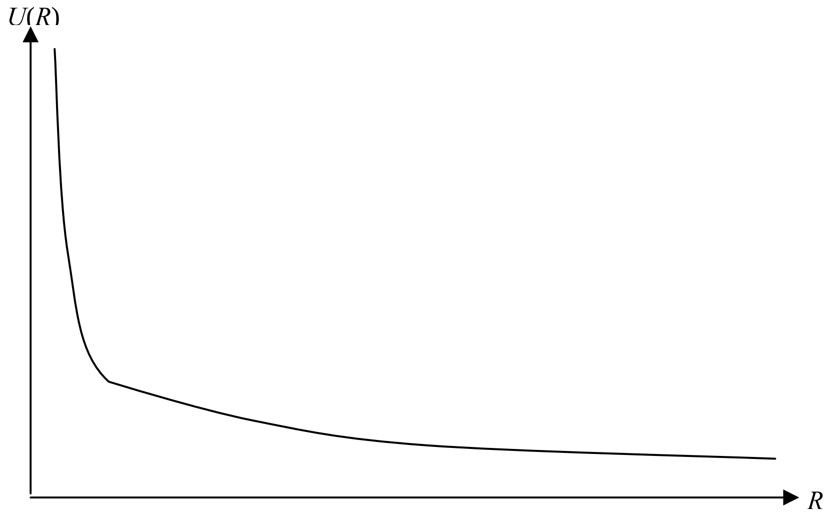

Fig. 4.5

4.3 Utility theory and risk aversion

As seen in the above, most of the people prefer to choose an alternative that is less risky although it may return a lesser pay-off, i.e. they prefer risk aversion. Although some individuals may be risk seekers, still some other can be risk-neutrals. These concepts of risk aversion, neutrality, or risk seeking may further be explained with the help of **marginal utility**.

Marginal utility is the utility drawn from consumption of an additional unit.

Risk seekers are not frequently found individuals, although in some situations, it may become unavoidable to take risk. e.g., a person in dire need of money has the only course of gambling open before him. In contrast, risk neutrality is fairly common, particularly when the amount at stake is not very high. A risk neutral individual will choose an alternative that will maximize his expected, monetary pay-off. However, a risk neutral individual may become risk averse if the amounts at stake are very high and a failure of decision may become very costly.

Most individuals are, however, risk averse. In order to ensure at least minimum return, they will go for the least risky alternative.

Consider the following graph:

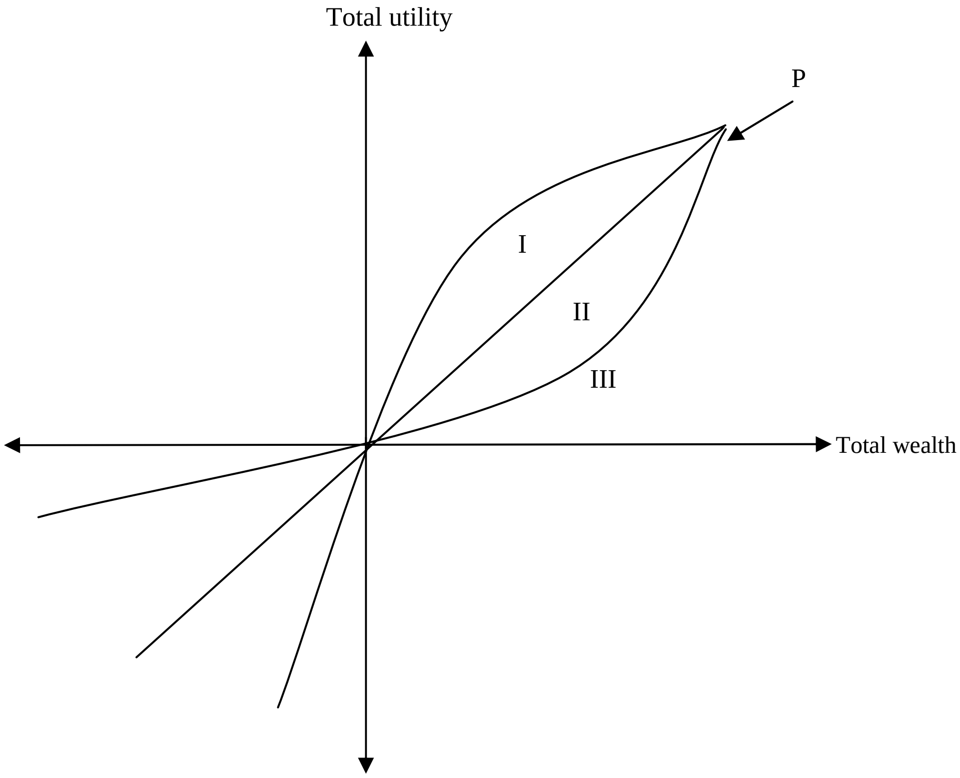

Fig. 4.6

Representing money income along X-axis and utility along Y-axis, the three curves are representing utilities of three individuals, say X, (curve I), Y (curve II) and Z (curve III).

For wealth 0, we assign a utility 0. Let the upper limit of utility be 100 utils. This point is being represented by P. As we know, the utility curve I, which is concave downwards, is of a risk averter. The slope of the utility curve is declining as the wealth is increasing, i.e., the total utility is increasing at a decreasing rate, or, in other words, the marginal utility is diminishing.

Curve II is that of a risk-neutral. In this case, the utility is increasing at constant rate with wealth. The marginal utility of this individual is constant.

Curve III is that of a risk seeker or a gambler. The curve is convex upwards, i.e., the total utility is increasing at an increasing rate i.e. the marginal utility of this individual is increasing.

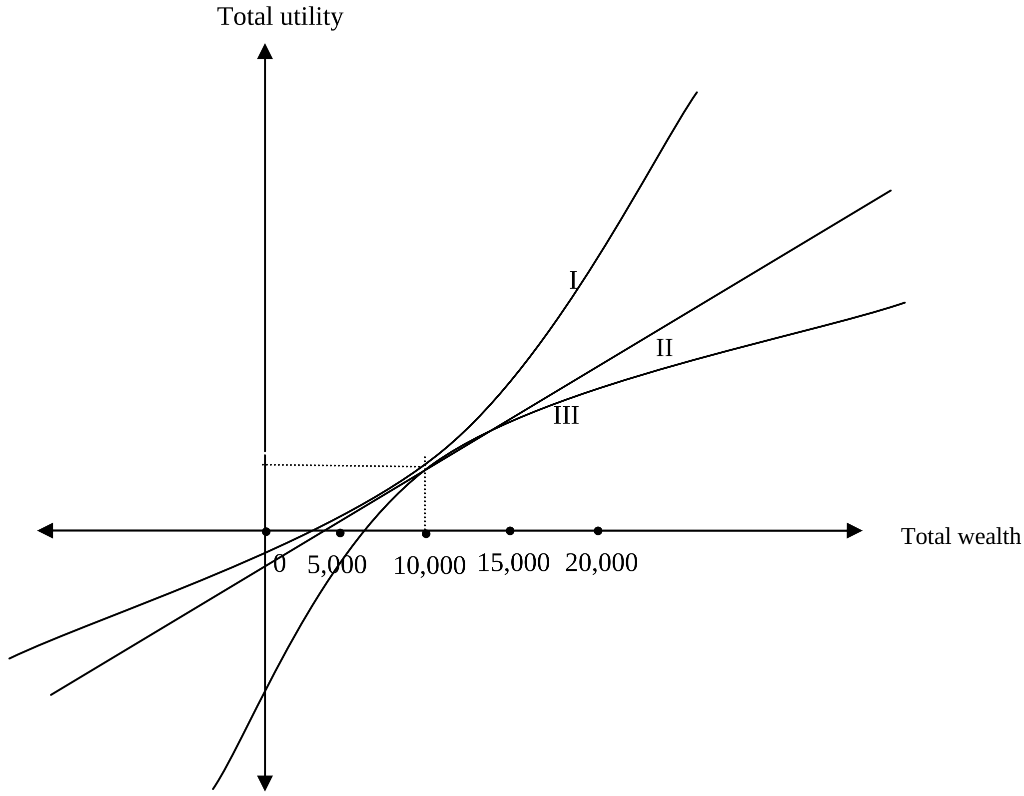

Fig. 4.7

Risk is inversely related to the marginal utility. As the risk increases, the utility decreases. Since most people are averse to risk, so their utility curves are convex or facing downwards. As we discussed in case of lottery, the extent of risk seeking or risk aversion is related to the amount at stake, i.e., the probability of winning the lottery game was same for the two gambles still most of the people would opt for an alternative involving lesser amount at stake. Since the boldness involving Rs. 2,000 is more than the boldness involving Rs. 10,000, this means the (marginal) utility of earning Rs. 10, 00,000 is more than the (marginal) utility of earning Rs. 50, 00,000. Thus it is possible to have same utility at some levels for risk averters, neutrals as well as seekers; the curves will become different at higher levels.

In case of individual Z, the utility from gaining second Rs. 10,000 (i.e., from Rs. 10,000 to Rs. 20,000) is less than the utility of loosing first Rs. 10,000.

This can, alternatively be stated as; the marginal utility decreases with money. The more we have, the less we are affected by small losses or small gains.

This argument can be proved mathematically also:

Consider a 50-50 bet of winning Rs. 10,000 or loosing the same amount as a result of a gamble game. In this case the expected value of money is

$$
\begin{aligned}
E(M) &= 0.5(10,000) + 0.5(-10,000) \\
&= 5,000 - 5,000 \\
&= 0
\end{aligned}
$$

Since the probability of winning the game is same as the probability of loosing it so the game is a fair game.

Now, consider the expected utility of a risk averter. If the risk averter (Z) looses Rs. 10,000, he looses two utils of utility, whereas if he wins, he only gains one util of utility. So in case of a fair game, the expected utility is given by

$$
\begin{aligned}
E(U) &= 0.5(1) + 0.5(-2) \\
&= 0.5 - 1 \\
&= -0.5
\end{aligned}
$$

So a risk averter may refuse a fair game also. For the same reason, a risk-averting manager may deny a project with a positive expected value.

Consider the following utility curve:

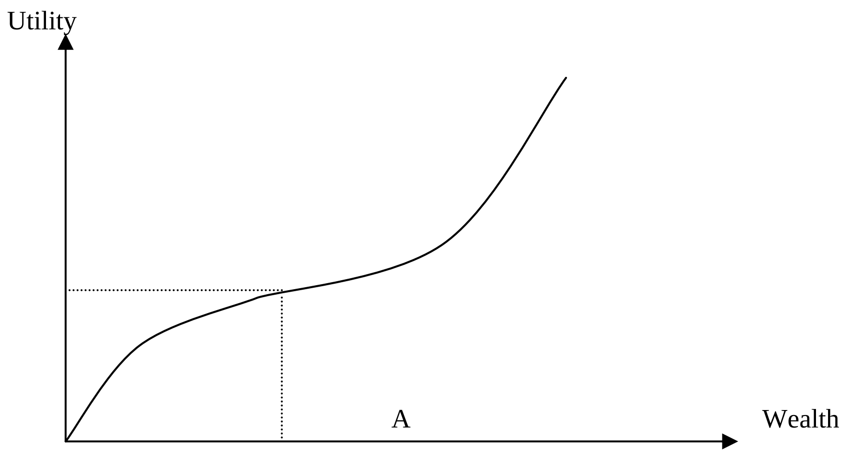

Fig. 4.8

This curve is concave downwards in the beginning and it becomes convex later. This curve belongs to a person who, at times, avoids taking risk and at some other times, may choose to gamble. At the point of inflection, although the person wants to insure himself but he will not mind taking the risks in investments, which will yield him larger returns. If his wealth goes down the point A, the person will avoid taking the risk. Thus the point is a measure of tolerance for such an individual for taking risk.

4.4 Utility and decision making

We know that the decision making is choosing an (most) admissible action or decision in a given frame work of states of nature and set of actions (decisions). Now we define the problem of decision making in terms of utilities of pay-offs of actions.

Let

(i) A is the set of possible actions which a decision maker can take.

(ii) Θ is the set of all states of nature.

(iii) $U(a, \theta)$ is the utility of an action a in A and a state of nature θ in Θ.

Then an action a^* is an optimal action (should be chosen) iff

$$EU(a^*) \;\geq\; EU(a) \quad \forall a \in \mathsf{A} \tag{4.1}$$

where

$$EU(a) = \begin{cases} \sum_{\theta \in \Theta} U(a,\theta)\, p(\theta)\ ; & \text{if } \theta \text{ is discrete} \\ \int_{\theta \in \Theta} U(a,\theta)\, p(\theta)\, d\theta; & \text{if } \theta \text{ is continuous} \end{cases} \qquad \forall a \in \mathsf{A}$$

Condition (4.1) is equivalent to the maximization of expected reward or minimization of expected loss, i.e., (4.1) is true iff

$$\begin{aligned} ER(a^*) &\geq ER(a) \\ & \qquad\qquad \forall a \in \mathsf{A} \\ EL(a^*) &\leq EL(a) \end{aligned}$$

We illustrate some examples to make the concept of utility clear.

Example 1: This is the famous oil drilling venture problem (variants of which have been considered through out this book).

A decision maker has obtained the drilling rights of a certain location. He has the following possible actions an optimal action to choose from:

(i) a_1: Drill alone with 100% interest.

(ii) a_2: Drill in partnership with 50% interest.

(iii) a_3: Sell the drilling rights for 25% of the net profit with no partnership in costs or losses.

(iv) a_4: Sell the drilling rights for Rs. 10, 00,000 plus 15% of the net profit with no partnership in costs or losses.

(v) a_5: Do not drill and sell the drilling rights for Rs. 50,00,000; and

(vi) a_6: Do not drill and retain the rights.

The possible states of nature consist of the quantity of oil which can be extracted from the location. Let θ be the random variable denoting the quantity of oil (in barrels) which can be possibly extracted. The quantity can be characterized with the help of a probability distribution. On the basis of other neighboring sights and geological information of the sights, the decision maker arrives at the following probability distribution:

$$P(\theta) = \begin{cases} 0.60\ ; & \text{for } \theta = 0 \\ 0.11; & \text{for } \theta = 1{,}00{,}000 \\ 0.15; & \text{for } \theta = 2{,}50{,}000 \\ 0.08; & \text{for } \theta = 7{,}00{,}000 \\ 0.05; & \text{for } \theta = 10{,}00{,}000 \\ 0.01; & \text{for } \theta = 15{,}00{,}000 \end{cases}$$

To determine the pay-offs for any possible combination of an action and a state of nature, he has made the following estimates:

(iii) The cost of drilling is Rs. 25,00,000; and

(iv) The profit per barrel of oil will be Rs. 50.

Then for any action a and state of nature θ, the pay-off or utility functions are given by

(i) $U(a_1, \theta) = 50\theta - 25{,}00{,}000$

(ii) $U(a_2, \theta) = 25\theta - 12{,}50{,}000$

(iii) $U(a_3, \theta) = 12.5\theta$

(iv) $U(a_4, \theta) = 7.5\theta + 10{,}00{,}000$

(v) $U(a_5, \theta) = 50{,}00{,}000$

(vi) $U(a_6, \theta) = 0$

On the basis of this information, we want to find the most optimal action for the decision maker.

The pay-off table for various actions is as follows:

Table 4.1

θ and $p(\theta)$ / a and $U(a, \theta)$	0 0.60	1,00,000 0.11	2,50,000 0.15	7,00,000 0.08	10,00,000 0.05	15,00,000 0.01
a_1 $50\theta - 25,00,000$	**-2500000**	2500000	10000000	32500000	47500000	**72500000**
a_2 $25\theta - 12,50,000$	-1250000	1250000	5000000	16250000	23750000	36250000
a_3 12.5θ	0	1250000	3125000	8750000	12500000	18750000
a_4 $7.5\theta + 10,00,000$	100000	850000	1975000	5350000	7600000	11350000
a_5 50,00,000	50,00,000	50,00,000	50,00,000	50,00,000	50,00,000	0
a_6 0	0	0	0	0	0	0

Clearly action a_6 is inadmissible. We calculate the expected pay-offs of the remaining actions:

Table 4.2

Action	Expected pay-offs $= \sum_{\theta} U(a,\theta)\, p(\theta)$
a_1	**5975000**
a_2	2987500
a_3	2118750
a_4	1371250
a_5	5000000
a_6	0

The decision maker should choose the first action, i.e. he should drill with 100% interest.

In computing the expected utility till now, we have been assuming that the utility function of the decision maker is (approximately) linear. However, if this is not the situation, then what happens?

Suppose that the individual is a risk avoider and considers various lotteries before reaching at a final decision. From table 4.1, we notice that the largest pay-off that he can receive is Rs. 7, 25, 00,000 and the smallest pay-off is -Rs. 25, 00,000. So he assigns U (7, 25, 00,000) = 100 and U (- 25, 00,000) = 0.

He then weighs the following two options for his risk preferences:

(i) Receive Rs. 0 with certainty.

(ii) Receive Rs. 7, 25, 00,000 with probability p and -Rs. 25, 00,000 with probability 1-p.

He finds that for p =0.30, he would go for the first alternative and for p = 0.60, he would go for the second alternative. If p = 0.40, then he would be indifferent to the two alternatives. This means in order for the decision maker to be indifferent his expected monetary pay-off should be

$$(0.40)(7,25,00,000)+(0.60)(-25,00,000) = 2,75,00,000$$

So he assigns U (0) = 40 Next, He weighs the following two options for his risk preferences:

(i) A return of Rs. 15, 00,000 with probability 0.6 or a return of Rs. 750,000 with probability 0.4.

(ii) A 20-80 bet of Rs. 4,75,00,000 or nothing.

Now, we calculate the expected pay-offs of the two alternatives:

(i) Expected pay-off of the first offer $= 0.6(1500000)+0.4(750000) = 1200000$

(ii) Expected pay-off of the second offer $= 0.20(0)+0.80(4,75,00,000) = 38000000$

Since the expected pay-off of the second alternative is higher than the expected pay-off of the second offer, the individual will go for the first alternative and he assigns U (475, 00,000) = 80.

Similarly after weighing several options he gets the following utility curve

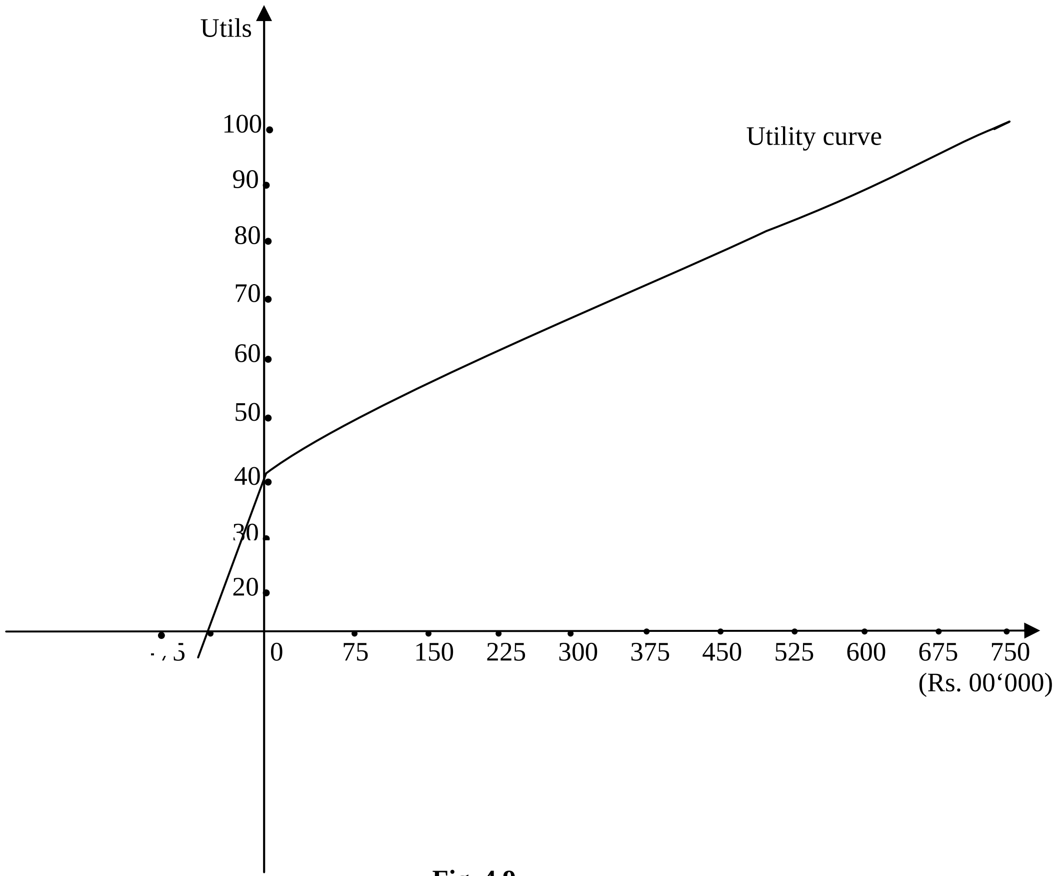

Fig. 4.9

From curve 4.9, we get the following table of utilities:

Table 4.3

θ and $p(\theta)$ / a and $U(a, \theta)$	0 0.60	1,00,000 0.11	2,50,000 0.15	7,00,000 0.08	10,00,000 0.05	15,00,000 0.01
a_1	**0**	62	53	68.5	80	**100**
a_2	28	53	81	55	61.5	70
a_3	40	53	68	50	53	58
a_4	41	42	43	46	50	51.5
a_5	81	81	81	81	81	81

The following table represents the expected utilities of the various actions that may be chosen by the decision maker:

Table 4.4

Action	Expected utility $= \sum_{\theta} U(a,\theta)p(\theta)$
a_1	25.25
a_2	42.955
a_3	**47.26**
a_4	42.365
a_5	81
a_6	0

Thus if the decision maker is going be maximizing his expected utility and is a risk avoider by nature, then he would choose action 3, viz. sell the drilling rights for 25% of the net profit with no partnership in costs or losses than action 1, viz. drill with 100% interest that he had chosen when he was deciding on the basis of monetary pay-offs. This is because of the risk avoiding nature of the decision maker that he is contented with a smaller certain pay-off than a larger uncertain pay-off.

Example 2: Consider the following utility function

$$U(a) = a^2 - 50a + 5; \quad a \geq 0$$

Where a is the pay-offs in Rs. ('000).

For this utility function, we would like to find the risk related behavior of the individual. Let us draw the graph of the curve to ascertain his risk attitude:

Table 4.5

a	$U(a)$
0	5
75	1880
150	15005
225	39380
300	75005
375	121880
450	180005
525	249380
600	330005

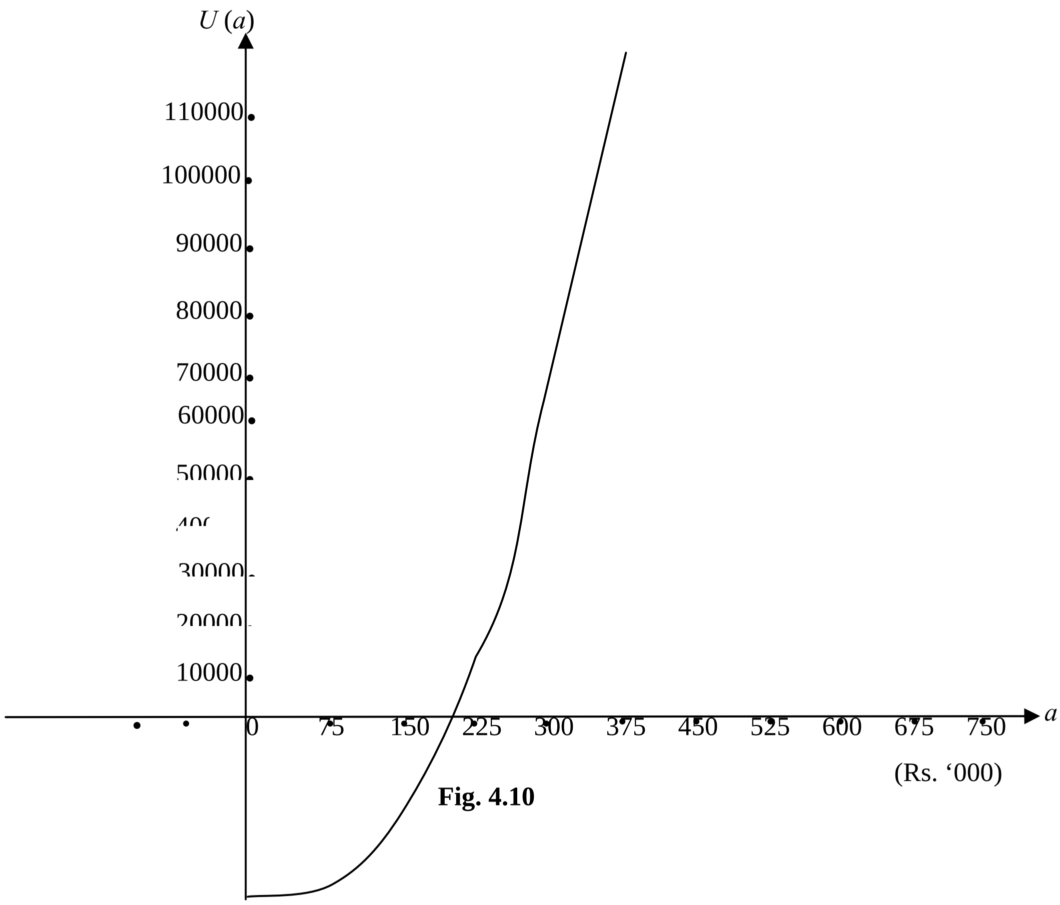

Fig. 4.10

As is evident from the curve drawn above, the individual is highly risk seeker as the steepness of the curve is rising sharply as the amount on stake is increasing.

Let us consider some bets and see the behavior of the individual towards those bets. One thing is for sure. For a risk seeker, the utility of pay-off is more than the monetary value of the pay-off. So if the risk seeker is choosing a bet on the basis of his monetary pay-off, he would definitely choose it on the basis of his utility.

(i) Bet I:

(a) A sure sum of Rs. 1, 00,000.

(b) A sum of Rs. 5, 00,000 with probability 0.6 or 0 with probability 0.4.

Expected pay-off of the second offer $= 0.6(500000) + 0.4(0) = 300000$

The individual will go for the second alternative as expected pay-off in this case is more than the sure sum of the first alternative.

Expected utility of the first offer $U(100000) = (100000)^2 - 50(100000) + 5 = 9995000005$

Also

$U(5,00,000) = 2.49975E+11;$ and $U(0) = 5$

$\therefore$

Expected utility of the second offer $= 0.6(2.49975E+11) + 0.4(5) = 1.49985E+11$

Again, the individual will go for the second alternative as expected utility in this case is more than the expected utility of the first alternative.

(ii) Bet II:

(a) A sure sum of Rs. 50, 00,000.

(b) A sum of Rs. 100, 00,000 with probability 0.6 or 0 with probability 0.4.

Again in this case

Expected pay-off of the second offer $= 0.6(1000000) + 0.4(0) = 600000$

The individual will go for the second alternative as expected pay-off in this case is more than the sure sum of the first alternative.

Expected utility of the first offer $U(500000) = (500000)^2 - 50(500000) + 5 = 2.49998E+13$

Also

$U(10,00,000) = 9.99995E+13$

; and $U(0) = 5$

$\therefore$

Expected utility of the second offer $= 0.6(9.99995E+13) + 0.4(5) = 5.99997E+13$

Again, the individual will go for the second alternative as expected utility in this case is more than the expected utility of the first alternative.

(iii) Bet III:

(a) A sure sum of Rs. 50, 00,000.

(b) A sum of Rs. 500, 00,000 with probability 0.1 or 0 with probability 0.9.

Expected pay-off of the second offer $= 0.1(50000000) + 0.9(0) = 5000000$

The individual is indifferent to the two alternatives as his expected pay-offs are the same in both the cases

$$\text{Expected utility of the first offer } U(5000000) = (5000000)^2 - 50(5000000) + 5 = 2.49998\text{E+13}$$

Also

$$U(50,00,000) = 2.49998\text{E+13}; \text{ and } U(0) = 5$$

$\therefore$

$$\text{Expected utility of the second offer} = 0.1(2.49998E+13) + 0.4(5) = 2.49998E+12$$

Again, the individual is indifferent to the two alternatives as his expected pay-offs are the same in both the cases.

(iv) Bet IV:

(a) A sum of Rs. 5, 00,000 with probability 0.8 or 50,000 with probability 0.2.

(b) A sum of Rs. 10, 00,000 with probability 0.25 or 2, 00,000 with probability 0.75.

$$\text{Expected pay-off of the first offer} = 0.8(500000) + 0.2(50000) = 410000$$

$$\text{Expected pay-off of the second offer} = 0.25(1000000) + 0.75(200000) = 400000$$

The individual will go for the first alternative as expected pay-off in this case is more than the expected pay-off of the second alternative.

$$U(5,00,000) = 1.9998\text{E+11}; \text{ and } U(50000) = 499500001$$

$\therefore$

$$\text{Expected utility of the first offer } U(5000000) = 0.8(1.9998\text{E+11}) + 0.2(499500001) = 2.0048\text{E+11}$$

Also

$$U(10,00,000) = 2.49988\text{E+11}; \text{ and } U(200000) = 29992500004$$

$\therefore$

$$\text{Expected utility of the second offer} = 0.25(2.49988E+11) + 0.75(29992500004) = 2.7998\text{E+11}$$

By utility criterion, the individual will go for the second alternative as expected utility in this case is more than the expected utility of the first alternative

Example 3: Consider the following utility function

$$U(a) = \sqrt{a} + \frac{1}{a}; \quad a > 0$$

Where a is the pay-offs in Rs. ('000).

For this utility function again, we would like to find the risk related behavior of the individual

Let us draw the graph of the curve to ascertain his risk attitude:

Table 4.6

a	$U(a)$
75	8.673587371
150	12.25411538
225	15.00444444
300	17.32384141
375	19.3675834
450	21.21542566
525	22.91478324
600	24.49656409
675	25.9822436
750	27.38746121
825	28.72402535
900	30.00111111

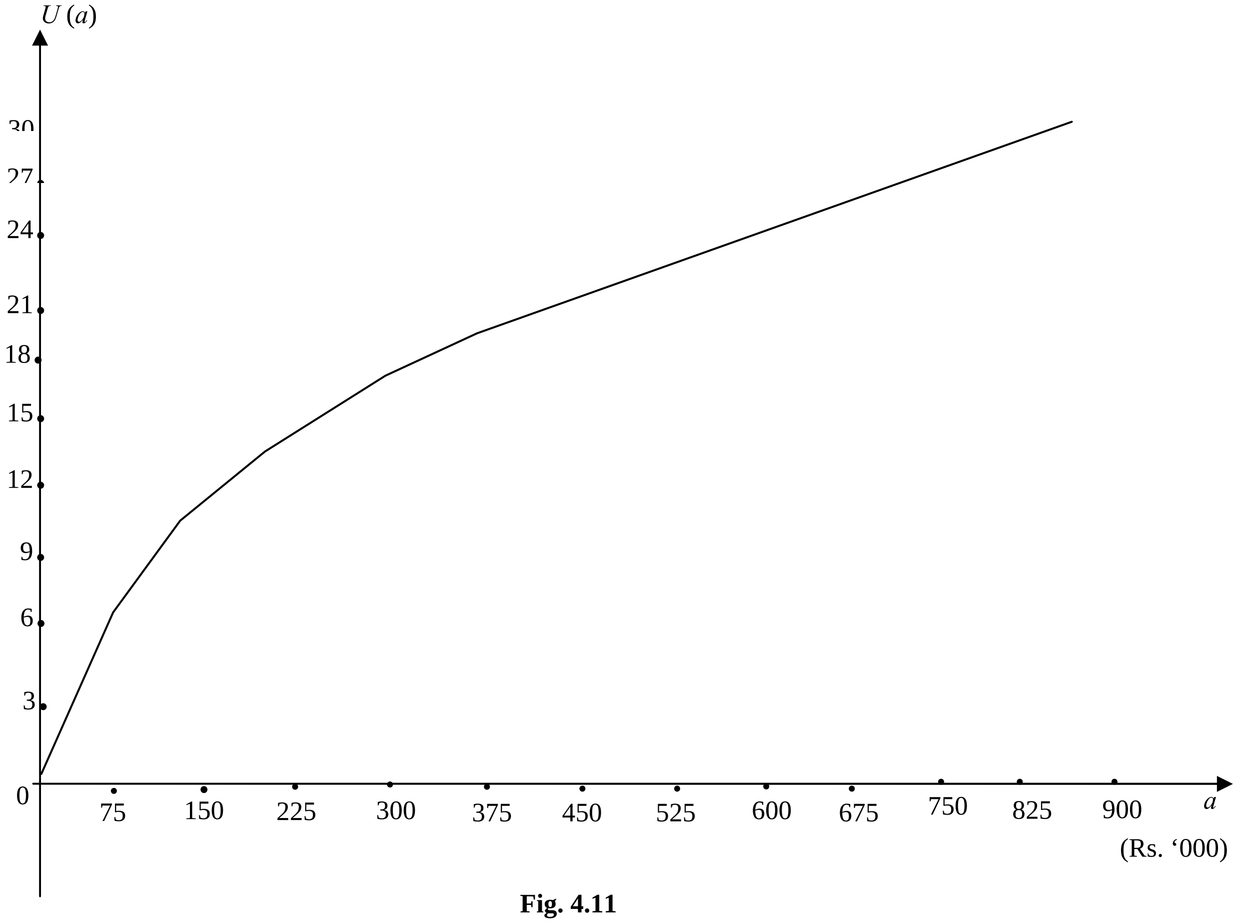

Fig. 4.11

As is evident from the curve drawn above, the individual is risk avoider as the the curve is rising facing downwards.

Again, let us consider some bets and see the behavior of the individual towards some bets.

(i) Bet I:

(a) A sure sum of Rs. 1, 00,000.

(b) A sum of Rs. 5, 00,000 with probability 0.6 or10000 with probability 0.4.

Expected pay-off of the second offer $= 0.6(500000) + 0.4(0) = 300000$

The individual will go for the second alternative as expected pay-off in this case is more than the sure sum of the first alternative.

Expected utility of the first offer $= U(100000) = 10.01$; and

Expected utility of the second offer $=0.6(22.36)+0.4(7.09)=16.25$

Again, the individual will go for the second alternative as expected utility in this case is more than the expected utility of the first alternative.

(ii) Bet II:

(a) A sure sum of Rs. 50, 00,000.

(b) A sum of Rs. 100, 00,000 with probability 0.6 or 750000 with probability 0.4.

Again in this case

$$\text{Expected pay-off of the second offer} = 0.6(1000000) + 0.4(0) = 600000$$

The individual will go for the second alternative as expected pay-off in this case is more than the sure sum of the first alternative.

$$\text{Expected utility of the first offer } U(500000) = 22.66; \text{ and}$$

$$\text{Expected utility of the second offer } = 22.47$$

The individual will go for the first alternative as expected utility in this case is more than the expected utility of the second alternative.

4.5 Utility curves and the managers

A risk averting person, as shown in his utility curve, is a biased person against losses. This observation is corroborated by the fact that the utility curve of this person falls very sharply when the expected pay-offs is negative (losses). This person, in capacity of a manager, would take a biased judgment against a risky project although it may involve attractive pay-off. For this person, the maximum utility will reach a maximum value after which all the more wealth will have same meaning for him. This policy will be against the interest of the company in the long run.

A risk neutral may also loose his enthusiasm for choosing somewhat riskier options after some time. But for a risk seeker, the utility will keep on increasing with the wealth and this person will have an ever-lasting temptation for taking risks in order to increase the business.

If a company wishes to expand its business, it must, at least at times, motivate its managers to risk taking.

Problems

1. SIS Technology is a company operating cyber cafés in a city. For an hourly fee of Rs. 12.00, the company provides access to a personal computer and Internet facility. The hourly variable cost to the company has been estimated to be Rs. 2.50. Now the company is planning to start a new café. The demand schedule for the computers (per hour) has been estimated as follows:

Table 4.7

Number of computers	15	16	17	18	19	20
Probability	0.10	0.15	0.25	0.30	0.15	0.15

In order to maximize its profit, how many PCs should be installed by the company? Find expected value of perfect information.

2. A BPO center in the city hires executives at an hourly rate of Rs. 175. The management of the center has estimated that the annual requirement of the executive hours is as follows

Table 4.8

Number of hours	10,000	12,000	15,000	18,000	20,000
Probability	0.20	0.25	0.30	0.15	0.05

If the revenue generated per executive hour is Rs. 210, find

(a) The number of executives that the center should hire.

(b) The expected value of perfect information.

It is known that the executives work 45 hours a week with a two-week annual vacation.

3. Contemporary Periodicals is a bookstore selling quarterly journals on current affairs. These journals are highly demanded by students preparing for various competitive examinations. A new journal costs Rs. 18 to the store and it fetches Rs. 26 to the store. In the second month of its publication, the journal would fetch only Rs. 20. However when a new addition comes, the left over stock can only be sold for Rs. 8 per journal. The owner of the store has estimated the following demand schedule for a new addition:

Table 4.9

Number of copies required	1000	1100	1200	1300	1400	1500
Probability	0.18	0.19	0.21	0.15	0.14	0.13

The order for new addition must be placed 20 days prior to its publication. Find the optimal number of copies to be ordered so as to maximize the profit of the store.

4. New India Times is a popular newspaper in a city. A newsstand sells this newspaper according to a normal distribution with mean 200 and standard deviation 50. The selling price of a copy is Rs. 2 and it costs Rs. 1.50 to the newsstand. Unsold copies can be sold for 20 paise per copy. In order to maximize its profit, how many copies should be ordered by the newsstand?

5. A firm has several investment proposals before it. The target rate of return of the firm is 10%, above which its utility rises very fast. Between a rate of 0% and 10%, the rise in utility is just marginal above 0, and below 0%, it declines very rapidly. If the amount that the firm wants to invest is Rs. 25, 00,000, draw the utility curve of the firm.

6. Consider the following information

   (a) An indifference between a sure sum of Rs. 20,000 or a 90:10 bet between a gain of Rs. 30,000 and a loss of Rs. 30,000.

   (b) An indifference between a sure sum of Rs. 10,000 or an 80:20 bet between a gain of Rs. 20,000 or nothing.

   (c) An indifference between a sure loss of Rs. 10,000 or a 40:60 bet between a loss of Rs. 20,000 or nothing

   If the sum Rs. 30,000 has utility 100 and –Rs. 20,000 utility 0, draw the utility curve. What can you say about the nature of the investor?

7. A Governmental funding agency is to sponsor NGOs working in the filed of rural employment. The maximum amount of sponsorship that can be offered is Rs. 15,00,000. The selection process of the NGOs which has been used till now, has classified the NGOs according to their performance as follows:

Table 4.10

Class	Proportion	Income generated (Rs.)
Poor	25%	-5,00,000
Average	50%	10,00,000
Good	20%	20,00,000
Excellent	5%	50,00,000

Now the sponsoring agency is planning to take help of a professional group, which would rate agencies (independent of ratings of the governmental agency) according to their efficiency. Three level of efficiency are C, B and A in increasing order. The following results have been obtained while relating the two classifications

Table 4.11

Classification of the professional group	Classification of the governmental agency			
	Poor	Average	Good	Excellent
A	0.10	0.10	0.40	0.60
B	0.20	0.80	0.40	0.30
C	0.70	0.10	0.20	0.10

(a) Using Bayes' theorem, determine whether or not, should the professional group be engaged?

(b) Does the hiring of the professional group really affect the true category of NGO?

(c) What is the maximum amount that can be paid to the professional group?

(d) If the professional group is to be paid Rs. 50,000, what should be the decision.

8. Consider the following utility functions:

(a) $U(a) = 100a - 5a^2$

(b) $U(a) = a^3 + a^2 - 50a; \quad a \geq 0$

(c) $U(a) = e^{-a} - 50a$

(d) $U(a) = \dfrac{1}{a^2} - \dfrac{50}{a} + 5$

In each of these cases, a represents the worth and $U(a)$ is the corresponding utility.

(i) Determine the risk behavior of the individuals whose utility functions are given above by drawing their utility curves.

(ii) Does the behavior remains same at all points or there are some points where the behavior is changing?

(iii) What conclusions do you draw after parts (i) and (ii)?

9. Consider the following utility functions:

(a) $U(a) = -100a - 5a^2$

(b) $U(a) = a^3 + a^2 + e^{-a}; \quad a \geq 0$

(c) $U(a) = -e^{-a}; a \geq 0$

(d) $U(a) = \dfrac{1}{a^2} + a^2$

In each of these cases, *a* represents the worth and *U* (*a*) is the corresponding utility.

(i) Determine the risk behavior of the individuals whose utility functions are given above by drawing their utility curves.

(ii) Consider the following bet for *p* = 0.25, 0.40, 0.60 and 0.74

  (i) A sure sum of Rs.50, 000; and

  (ii) A sum of Rs. 1, 00,000 with probability *p* and 10,000 with probability 1-*p*.

In each case, determine the preferred action.

10. An investor has found that his utilities for various amounts which he can invest are as follows: *U* (1, 00,000) = 100; *U* (50,000) = 80; *U* (25,000) = 60; *U* (0) = 50; *U* (-25,000) = 40; *U* (-50000) = 20; and *U* (-1, 00,000) =0.

He wants to invest Rs. 1, 00,000 for which he has the following possibilities:

  (i) The amount will be doubled with probability 0.45; and

  (ii) The amount will be halved with probability 0.55.

He is considering three investments with the same possibility of increasing or decreasing his amount.

The possible decisions which he might consider are as follows:

(i) Invest the whole amount in a single stock.

(ii) Invest equally in either of the two stocks.

(iii) Invest Rs. 90,000 equally in three stocks and save Rs. 10,000 with him.

(iv) Invest Rs. 60,000 equally in three stocks and save Rs. 40,000 with him.

(v) Does not invest anything and retain the amount with him. In this case he is not going to loose or gain anything.

Advise the optimal action to him taking into account

  (a) The expected monetary pay-off; and

  (b) The expected utility of the pay-offs.